# ENGINEERING MECHANICS

**SRI POLYTECHNIC , KOMAND**

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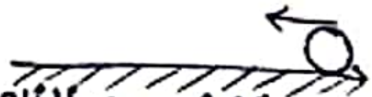
## Unit 2 :- Friction.

When a body moves or tends to move over another body a force opposing the motion develops at the contact surfaces. This force which opposes the movement or tendency of movement is called Friction Force (or) simple Friction.

<u>Types of Friction</u> :-



1) Solid (or) Dry Friction < Sliding friction / Rolling Friction

2) Viscous (or) fluid Friction

3) Non - viscous (or) Greases friction (boundary Friction).

<u>Dry (or) Solid Friction</u> :- the Friction that exist between perfectly clean and dry (unlubricated) Solid surfaces is called solid or dry dry friction. The two surfaces may be at rest (static friction) or one surface is moving on other surface is at rest (dynamic or kinetic Friction).

The Friction between two contacting surfaces at the point of sliding is called limiting Fric

The Friction that exist when one surface sides over the other is called sliding fricti-on.

the Friction that exist when one surface rolls over the other surface is called rolling Friction.

ii) viscous (or) fluid Friction:-

If a thick layer of oil or lubricant is introdu-ced between the two surfaces a film lubri-cant form on both the surfaces and there is no direct contract between the surfaces. the friction between two surfaces separated compl-etely by a film of lubricant is called vis-cous (or) fluid friction.

iii) Non viscous (or) Greasy (or) boundary Friction:-

The thin layer of an oil (or) lubricant Introduce-d between the two surfaces prevents the metal to contact and reduce the Friction the Friction exist between the two surfaces separated by an Extremely thin layer of oil is called non-viscous (or) Greasy friction.

## Laws of solid (or) dry friction:-

(i) Frictional Force always opposes the motion.

(ii) Frictional Force is independed of area in contact and depends upon nature and rough-ness of the surface.

(iii) Frictional force is propotional to the normal reaction between the surfaces

(iv) The static frictional force at any instant is equal to the force applied to the body

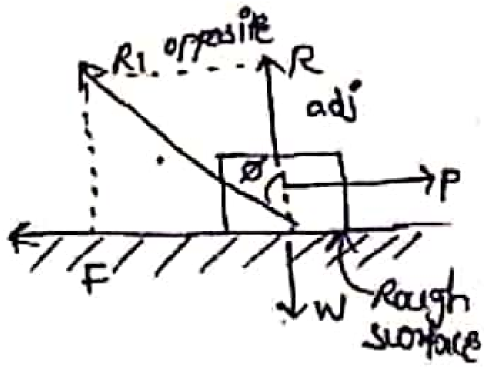
(v) For low speeds the kinetic frictional force is independed of the speed but increases slightly with increasing speed.

* **co-efficient of friction:-** The ratio of limiting force of friction to the normal reaction between two bodies is called co-efficient of friction it is denoted by a letter $\mu$

$$\text{Co-efficient of friction } \mu = \frac{\text{Limiting force of frictional force}(F)}{\text{Normal Reaction }(R)}$$

$$\boxed{F = \mu R}$$

*Imp Angle of Friction:- The angle between the resultant reaction (resultant of normal reaction and limiting force of friction) and the normal reaction. It is denoted by $\phi$




Let $R_1$ = Resultant Reaction

$R$ = Normal Reaction.

$F$ = limiting force of friction.

$\phi$ = Angle of friction

$$\mu = F/R$$

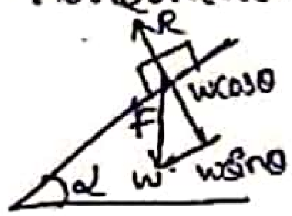
$$F = \mu R$$

$$\tan\phi = \frac{\text{opposite}}{\text{adj}} = F/R$$

$$= \frac{\mu R}{R}$$

$$\boxed{\tan\phi = \mu}$$

*Angle of Repose:- Let a body is resting on a rough inclined plane making an angle $\alpha$ with horizontal




If the body on the point of sliding the angle is θ.

$$F = W\sin\theta$$

$$R = W\cos\theta$$

$$\mu = F/R = \mu = \frac{W\sin\theta}{W\cos\theta}$$

$$= \tan\theta$$

$$\underline{\mu = \tan\phi}$$

$\tan\phi = \tan\theta$    $\theta$ is called angle of repose
$\theta = \phi$    and defined as maximum angle of the inclined plane at which a body.

## (1) Equilibrium of a body on horizontal plane:

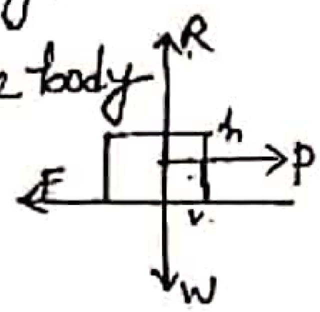
A body of weight 'w' rest on a horizontal sur-face, let $\mu$ be the co-efficient of friction between the body and horizontal surface. the least force p required to move the body in its directions

(i) Force is horizontal.

(ii) Force acts at given angle $\theta$

(1)* Force is applied horizontally:-

least force required to move the body




$P = F$ horizontal

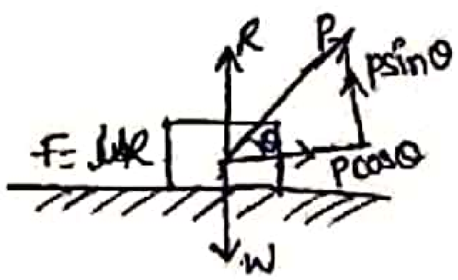
$F = \mu R$

$P = \mu R.$

vertical $R = W$

Force is applied act on $\theta$ with the horizontal:-

resolving forces horizontally




$F = P\cos\theta$

$\mu R = P\cos\theta \rightarrow (1)$

Resolving forces vertically.

$W = R + P\sin\theta$

$$R = W - P\sin\theta \longrightarrow (2)$$

By eqn (1) & (2)

$$\mu(W - P\sin\theta) = P\cos\theta$$

$$\mu W - \mu P\sin\theta = P\cos\theta$$

$$P(\mu\sin\theta + \cos\theta) = \mu W$$

$$P = \frac{\mu W}{\mu\sin\theta + \cos\theta}$$

$$\tan\phi = \mu$$

$$= \frac{\tan\phi\, W}{\tan\phi\sin\theta + \cos\theta}$$

multiply numerated & denominator with $\cos\phi$

$$= \frac{\sin\phi\, W}{\cos\phi\sin\theta + \cos\theta\cos\phi}$$

$$= \frac{\sin\phi\, W}{\cos(\theta - \phi)}$$

If the value $p$ is less

$$\tan\theta = \phi$$

$$P = \sin\phi\, W$$

$$\boxed{P = \sin\theta W}$$

(1) A small block of weight 50N is resting on a rough horizontal surface. The co-efficient of friction between the block and surface being 0.6. Find the least force which act as on the block at an angle of 60° with the horizontal will ~~...~~ cause the block to side

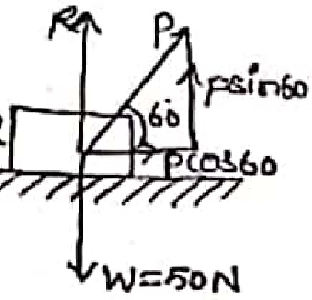
weight $(W) = 50N$

weight $(W) = 50N$

coefficent of friction $(\mu) = 0.6$

$\theta = 60^\circ$

$P = ?$




Resolving forces horizontally $F = \mu R$

$F = P\cos 60$

$0.6 \times R = P\cos 60$

$0.6 \times R = P \times 0.5 \rightarrow ①$

Resolving the forces vertically

$W = R + P\sin 60$

$50 = R + P\sin 60 \rightarrow ②$

$R = 50 - P\sin 60 \rightarrow ③$

$0.6 \times (50 - P\sin 60) = P\cos 60$

$0.6 \times 50 = P(0.6\sin 60 + \cos 60)$

$$P = \frac{0.6 \times 50}{(0.6\sin 60 + \cos 60)} = 57.44N$$

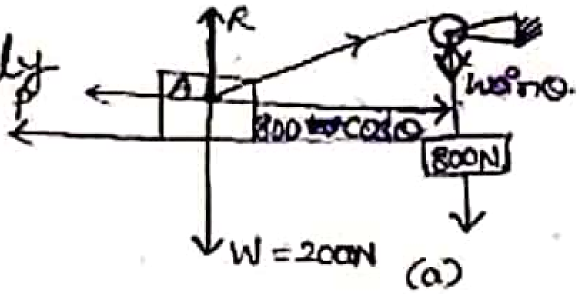
*I

② A block 'A' shown in figure weighs 2000N The Cord attached to 'A' pass through a frictionles, pulley and supports a weight equal to 800N. The value of coefficient of friction is 0.35 Slove the horizontal force P.

(i) If the motion is impending towards the left?

(ii) If the motion is impending towards right?

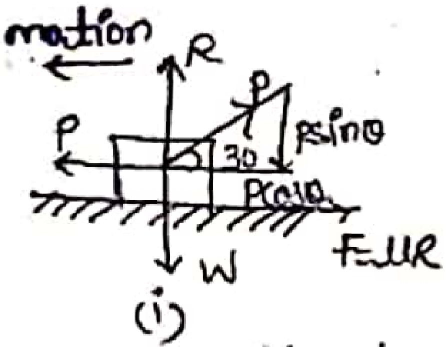
Resolving the force vertically

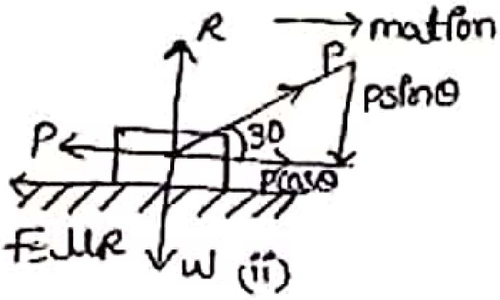



$R = W + 800\sin 30$

$= 2000 + 400$

$= 2400\,N$




Frictional force $(F) = \mu R$

$= 0.35 \times 2400$

$= 840\,N$

(i) motion impending towards left.

$P = F + \cos\theta \times 800$

$= 840 + 800 \times \cos 30$

$= 1532.82\,N$

(ii) motion impending towards right

$P + F = 800\cos\theta$

$P = 800\cos\theta - F$

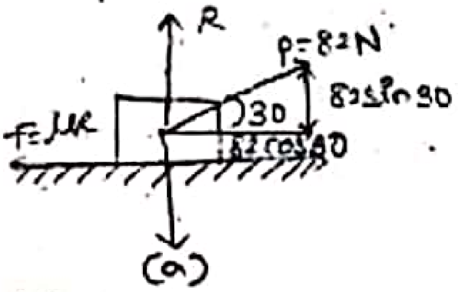
$= 800\cos 30 - 840$

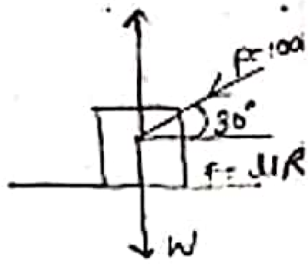
$= -147.17\,N$

$= 147.17\,N$ (toward of left)

A body is resting on rough horizontal plane required a pull of 82N inclined at 30° to the plane just move it. It was found that push of 100N inclined at 30° to the plane just moved the body. Determine weight of the body and the coefficient of friction

Sol:-




(i) force (P)=82

$\theta = 30°$

Resolving forces horizontally

$F = 82\cos30$

$\mu \times R = 82\cos30 \rightarrow (1)$

$= 71.012$

Resolving forces vertically

$W = R + 82\sin30$

$R = W - 82\sin30$

$R = W - 41$

$\mu \times (W-41) = 71.02 \rightarrow (3)$

(ii) Force (P) = 100N

$\theta = 30°$

Resolving forces horizontally

$F = 100\cos30$

$= 100 \times 0.866$

$\mu R = 86.6 \rightarrow (4)$

Resolving forces vertically

$R = W + 100\sin30$

$R = W + 50 \rightarrow (5)$

$$\mu(W+50) = 86.6 \rightarrow ⑥$$

$$\mu(W-41) = 71.02 \rightarrow ③$$

$$\mu(W+50) = 86.6 \rightarrow ⑥$$

$$\mu W + 50\mu = 86.6$$

$$\mu W - 41\mu = 71.02$$

$$91\mu = 15.58$$

$$\mu = \frac{15.58}{91} = 0.171$$

$$0.171(W-41) = 71.02$$

$$W = \frac{71.02}{0.171} + 41 = 455.5\,N.$$

(4a) A wooden block weighing 30N is placed on horizontal plane. A horizontal force of 12N is applied and the block is on the point of moving?

(i) co-efficient of friction.

(ii) Angle of friction.

(iii) The Resultant reactions.

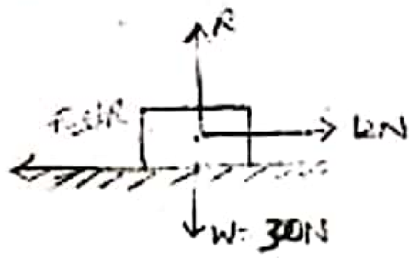
$W = 30N$

$P = 12N$

$\mu = ?$

$\phi$ ?

$R = ?$




Resolving forces horizontally

$$F = 12N$$

$$\mu R = 12N \rightarrow ①$$

Resolving vertical forces

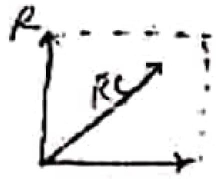
$$R = W$$

$$= 30 \rightarrow ②$$

$\mu = \frac{12}{R}$

$= \frac{12}{30}$

$= 0.4$




$\tan\phi = \mu$

(ii) $\phi = \tan^{-1}(0.4)$

$= 21.8^{\circ}$

(iii) $Rc = \sqrt{R^2 + P^2}$

$= \sqrt{30^2 + 12^2}$

$= 32.31\,N$.

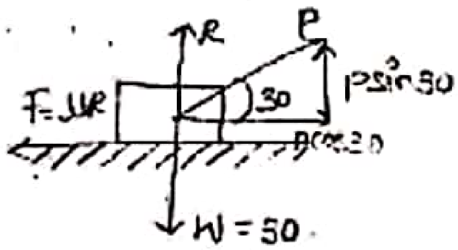
b) a wooden block. weight 80N is placed a horizonta-l plane where the coefficient of friction is 0.25 Find the force that show of 30° with the horizontal to in the condition of equilibrium ?

$W = 80N$

$\mu = 0.25$

$\theta = 30^{\circ}$

$P = ?$




Resloving forces horizontally

$F = P\cos 30$

$\mu \times R = P\cos 30 \longrightarrow (1)$

Resalving forces vertically

$W = R + P\sin 30$

$R = W - P\sin 30 \longrightarrow (2)$

$0.25(W - P\sin 30) = P\cos 30$

$\mu W - \mu P\sin 30 = P\cos 30$

$\mu W = P(\cos 30 + 0.25 \times \sin 30)$

$$P = \frac{0.25 \times 80}{(\cos 30 + 0.25 + \sin 30)}$$

$= 23.211$

**equilibrium of a body on a rough inclined surface:**

If the inclination of a plane is more than angle of repose. i.e. body will need extreme force to maintain equilibrium the magnitude of force in the following directions will be considered.

(i) The force p is parallel to the plane

(ii) The force p is horizontal

(iii) The force p is inclined at angle θ with the plane.

(i)a) The force is parallel to the plane and the body tends to slide down:-

Consider a body of weight W lying on rough inclined plane 'θ' be the inclination of the plane ($\theta > \alpha$) and p be the force applied along the plane.

The forces keeping the body under the Equilibrium as shown in figure.

Resolving forces normal to the inclined plane.

$R = W \cos\alpha$

resolving forces along the inclined plane.

$P = W \sin\alpha + F$

$= W\sin\alpha + \mu R \longrightarrow ②$

from Eqn ① & ②

$P = W\sin\alpha + \mu (W\cos\alpha)$

$= W\sin\alpha + \tan\phi\,(W\cos\alpha)$

$= W(\sin\alpha + \tan\phi\cos\alpha)$

$= W\left(\sin\alpha + \frac{\sin\phi}{\cos\phi}\cos\alpha\right)$

$= W\left(\frac{\sin\alpha\cos\phi + \sin\phi\cos\alpha}{\cos\phi}\right)$

$$\boxed{P = W\left[\frac{\sin(\alpha+\phi)}{\cos\phi}\right]}$$

b) The force is parallel to the plane, the body tends to move up the plane.

when the body tends to move up the plane the force of friction is acting down the plane i.e. in the direction opposite to the applied force. The various forces keeping the body under equi-librium as shown in the figure.

⇒ Resolving forces Normal to the plane, $R = W\cos\alpha \rightarrow ①$

Resolving forces along inclined plane.

$P = W\sin\alpha + F$

$= W\sin\alpha + \mu R \rightarrow ②$

from Eq ① & ②

$P = W\sin\alpha + \mu(W\cos\alpha)$

$= W\sin\alpha + \tan\phi\,(W\cos\alpha)$

$= W(\sin\alpha + \tan\phi\cos\alpha)$

$= W\left(\sin\alpha + \frac{\sin\phi}{\cos\phi}\cos\alpha\right)$

$= \frac{W(\sin\alpha\cos\phi + \sin\phi\cos\alpha)}{\cos\phi}$ $\Rightarrow P = W\left(\frac{\sin(\alpha+\phi)}{\cos\phi}\right)$

4)a) The force is horizontal (i.e, parallel to the base) and body tends to slide down:-

- consider a body weight "w" lying on a rough inclined plane. Let the force 'p' be applied horizontal to prevent the body sliding down. Since the body tends to slide down a frictional force act up on the plane.

The various forces keeping the body under equilibrium.


Resolving the forces normal to the plane.

$$R = W\cos\alpha + p\sin\alpha \longrightarrow ①$$

Resolving forces along the plane.

$$p\cos\alpha + F = w\sin\alpha \longrightarrow ②$$

$$P\cos\alpha = w\sin\alpha - \mu R \longrightarrow ③$$

From eqn ① & ③

$$p\cos\alpha = w\sin\alpha - \mu(w\cos\alpha + p\sin\alpha)$$

$$P(\cos\alpha + \mu\sin\alpha) = w(\sin\alpha - \mu\cos\alpha)$$

$$P(\cos\alpha + \tan\phi\sin\alpha) = w(\sin\alpha - \tan\phi\cos\alpha)$$

$$P\left(\cos\alpha + \frac{\sin\phi}{\cos\phi}\sin\alpha\right) = w\left(\sin\alpha - \frac{\sin\phi}{\cos\phi} \times \cos\alpha\right)$$

$$P\left[\frac{\cos\alpha\cos\phi + \sin\phi\sin\alpha}{\cos\phi}\right] = w\left[\frac{\sin\alpha\cos\phi - \sin\phi\cos\alpha}{\cos\phi}\right]$$

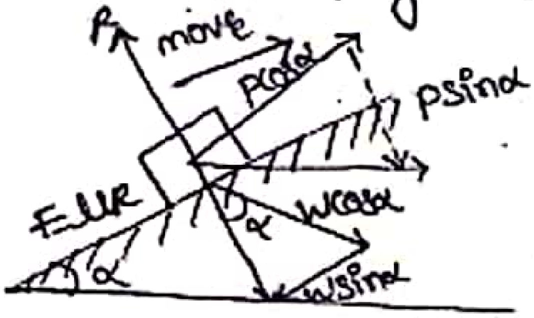
$$P\cos(\alpha - \phi) = w\sin(\alpha - \phi)$$

$$\boxed{P = w\tan(\alpha - \phi)}$$

(b) The Force is horizontal the body tends to move up the plane :-

when the body tends to move up the plane, the force of friction acts down the plane. Let force P applied horizontally to move the body up the plane.

The various forces keeping the body under equilibrium.




Resolving the forces Normal to the plane.

$$R = W\cos\alpha + P\sin\alpha \rightarrow (1)$$

Resolving fores along the plane.

$$P\cos\alpha = W\sin\alpha + F$$

$$P\cos\alpha = W\sin\alpha + \mu R \rightarrow (2)$$

from Eqn (1) & (2)

$$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha)$$

$$P(\cos\alpha - \mu\sin\alpha) = W(\sin\alpha + \mu\cos\alpha)$$

$$P(\cos\alpha - \tan\phi\sin\alpha) = W(\sin\alpha + \tan\phi\cos\alpha)$$

$$P\left(\cos\alpha - \frac{\sin\phi}{\cos\phi}\sin\alpha\right) = \left[W\sin\alpha + \frac{\sin\phi}{\cos\phi}\cos\alpha\right)$$

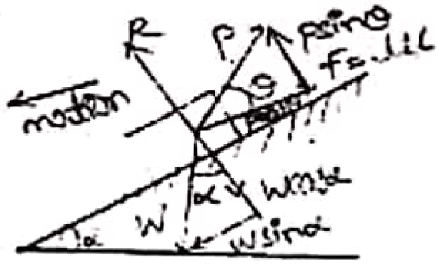
$$P\left[\frac{\cos\alpha\cos\phi - \sin\phi\sin\alpha}{\cos\phi}\right] = W\left[\frac{\sin\alpha\cos\phi + \sin\phi\cos\alpha}{\cos\phi}\right]$$

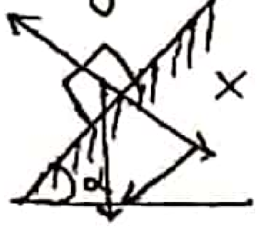
$$P\cos(\alpha+\phi) = W\sin(\alpha+\phi)$$

$$\boxed{P = W\tan(\alpha+\phi)}$$

(iii) the force make an angle '$\theta$' with the plane and the body tends to slide down:-

. Consider a body weigh 'w' lying on a rough inclined plane. Let the force 'p' applied at an angle '$\theta$' with inclined plane when the body tends to slide down the frictional force act up on the plane the various forces keeping the body under equilibrium.




∴ Resolving forces Normal to the plane.

$$R = W\cos\alpha - P\sin\theta \rightarrow ①$$

Resolving forces along the plane

$$P\cos\theta + F = W\sin\alpha \rightarrow ②$$

$$P\cos\theta = W\sin\alpha - \mu R$$

from ① & ②

$$P\cos\theta = W\sin\alpha - \mu(W\cos\alpha - P\sin\theta)$$

$$P(\cos\theta - \mu\sin\theta) = W(\sin\alpha - \mu\cos\alpha)$$

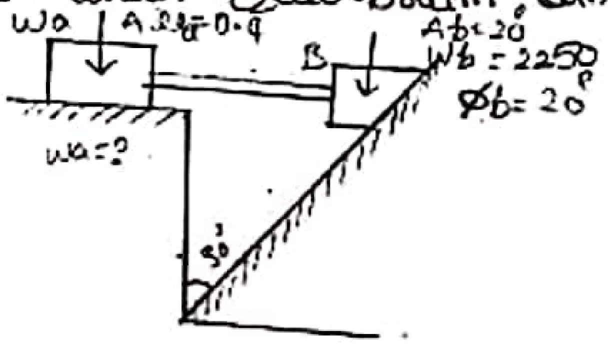
$$P(\cos\theta - \tan\phi\sin\theta) = W(\sin\alpha - \tan\phi\cos\alpha)$$

$$P\left(\frac{\cos\theta\cos\phi - \sin\phi\sin\theta}{\cos\phi}\right) = W\left[\frac{\sin\alpha\cos\phi - \sin\phi\cos\alpha}{\cos\phi}\right]$$

$$P\cos(\theta+\phi) = W\sin(\alpha-\phi)$$

$$P = \frac{W(\sin(\alpha - \phi))}{\cos(\theta + \phi)}.$$

Two blocks A and B are connected by a horizontal rod and supported on two rough planes shown in fig the coefficient of friction for block A is 0.4 the angle of friction for the blocks on the inclined plane is $\phi = 20°$ find the smallest weight of the block 'A' for which equilibrium can exist?

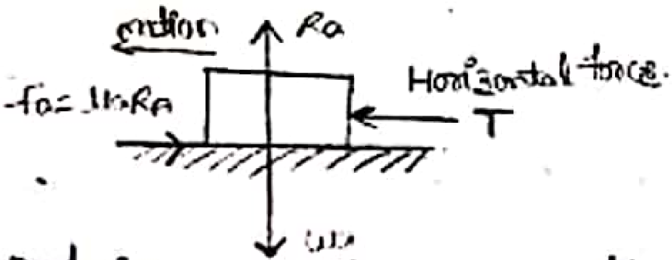



soln:-

$\mu a = 0.4$ $\qquad$ $Wb = 2250\,N$

$\phi b = 20°$ $\qquad$ $Wa = ?$

$\mu b = \tan 20 = 0.36$




Resolving vertical components.

$Ra = Wa \rightarrow ①$

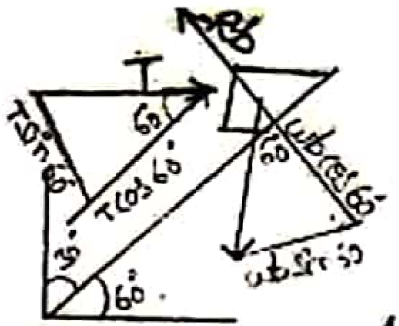
Resolving horizontal components.

$Fa = T$

$\mu a Ra = T$

$0.4 \times Ra = T \rightarrow ②$

$0.4 \times Wa = T$




Resloving force normal to the inclined plane.

$Rb = wb\cos60 + T\sin60$

$= 2250 \times 0.5 + T \times 0.866$

$= 1125 + 0.866T \rightarrow (3)$

Resolving forces along the plane.

$Fb + T\cos60 = wb\sin60$

$Fb = wb\sin60 - T\cos60$

$= 2250 \times 0.866 - T \times 0.5$

$= 1948.55 - 0.5T$

$\mu bRb = 1948.55 - 0.5T$

$0.36Rb = 1948.55 - 0.5T \rightarrow (4)$

Eqn (3) multiply with 0.36

$0.36Rb = 850.73 + 0.811$

$0.36Rb = 1948.55 - 0.5T$

$0.36Rb = 405 - 0.811$

$1543.55 - 0.811T$

$T = \frac{1543.55}{0.811}$

$= 1903.26N$ ,,

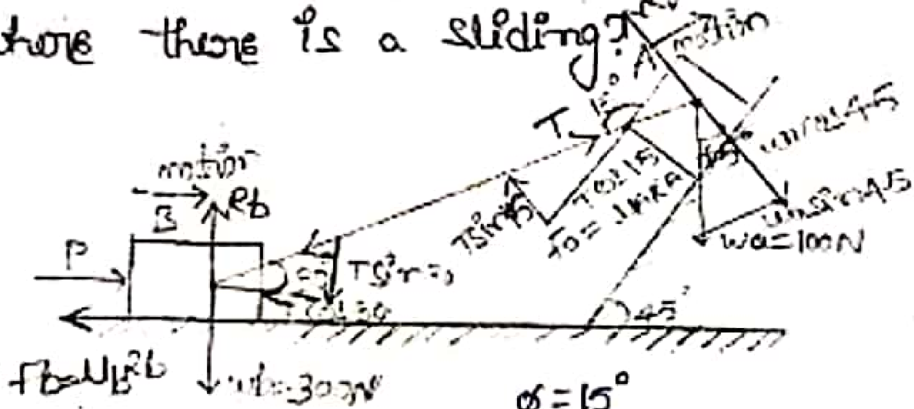
$0.4 \times wa = T$

$wa = \frac{1903.26}{0.4}$

$= 4758.15N$

A block A weighing 100N rests on a rough inclined plane whose inclination to the horizontal 45°. The block is connected to another block B weighing 300N resting on a rough horizontal plane, by a weightless rigid bar inclined at 30° to the horizontal. Find the horizontal force required to be applied to the block B to just move the block A in upward direction.

Assume angle of limiting friction as 15° at all surfaces where there is a sliding.




$\phi = 15°$

$\mu = \tan 15 = 0.25.$

considering block B

Resolving forces horizontally

$$P = T\cos 30 + F_b$$

$$= T \times 0.866 + 0.26 \times R_b \rightarrow (1)$$

Resolving forces vertically.

$$R_b = T\sin 30 + W_b$$

$$= T \times 0.5 + 300N \rightarrow (2)$$

from Eq$^n$ (1) & (2)

$$P = T \times 0.866 + 0.26 \times [T \times 0.5 + 300]$$

$$P = [T \times 0.866 + 0.26 \times T0.5] + [0.26(\times T \times 0.5 + 300]$$

$$P = 78 + 0.996T$$

$$P \simeq = 78 + T$$

consider block A

Resolving forces along the plane

$T\cos 15 = W_a \sin 45 + F_a$

$0.96T = 100 \times 0.707 + 0.26 R_a \longrightarrow (3)$

Resolving forces normal to the inclined plane.

$R_a + T\sin 45 = W_a \cos 45$

$R_a = 100 \times 0.707 - 0.259 T \longrightarrow (4)$

from Eqn (3) & (4)

$0.268 R_a = 18.94 - 0.069T$

$(-)\ 0.26 R_a = -70.7 + 0.96T$

$\cancel{0.26R} = 89.64 - 1.02T$

$T = \dfrac{89.64}{1.02}$

$= 87.88N$

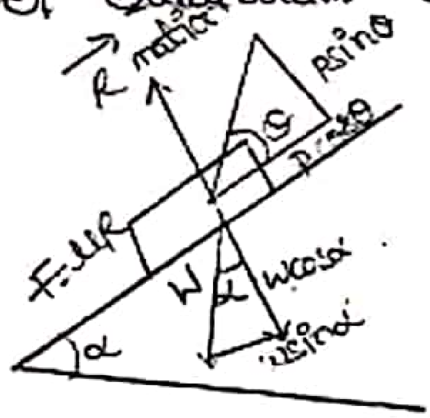
$P = 80.4 + T$

$P = 78 + T$

$= 78 + 87.88$

$= 165.88N.$

the force is applied at an angle θ with the pll-ne and the body tends to move up the plane. When the body is on the point of sliding up the plane. the frictional force acts down the plane. The system of force keeping the body under equilibrium as shown in fig.




Resolving the forces ⊥ to plane.

$R = W\cos\alpha - P\sin\theta \rightarrow (1)$

Resolving the forces along the plane.

$P\cos\theta = W\sin\alpha + F$

$P\cos\theta = W\sin\alpha + \mu R \rightarrow (2)$

from Eqⁿ (1) & (2)

$P\cos\theta = W\sin\alpha + \mu(W\cos\alpha - P\sin\theta)$

$P(\cos\theta + \mu\sin\theta) = W(\sin + \mu\cos\alpha)$

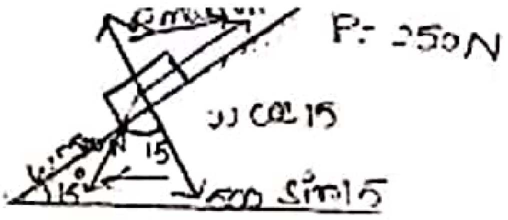
$P(\cos\theta + \tan\phi\sin\theta) = W(\sin\alpha + \tan\phi\cos\alpha)$

$P(\cos\theta\cos\phi + \sin\phi\sin\theta) = W(\sin\alpha\cos\phi + \sin\phi\cos\alpha)$

$P(\cos(\theta-\phi)) = W(\sin(\alpha+\phi))$

$\therefore P = W \dfrac{\sin(\alpha+\phi)}{\cos(\theta-\phi)}$.

A force of 250 N pulls a body of wt 500 N up an inclined plane, the force being applied ||el to it find the coefficient of friction




$P = 250N$
$W = 500N$
$\alpha = 15$
$\mu = ?$

11er to 15 fig

Resolving force normal to the plane

$R = W\cos 15$
$= 500\cos 15$
$= 500 \times 0.966$
$R = 483N$

$F = \mu R$
$\mu = F/R$
$= \frac{120.5}{483}$
$= 0.249$

Resolving forces along the plane.

$P = 500\sin 15 + F$

$F = P - 500\sin 15$
$= 250 - 500 \times 0.259 = 120.5N$

A body of wt 1000N is to be pulled up an inclined plane of angle 20; Coefficient of friction body & plane is 0.28 find the effect required

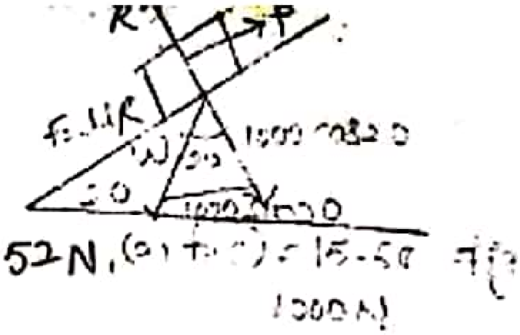
a) when it is 11el to plane.
b) " " " 11el to base
c) " " inclined to plane at 10.

sol:- $W = 1000N$
$\alpha = 20$
$\mu = 0.28$
$P = ?$
$\tan\phi = \mu$
$\phi = \tan^{-1}\mu$
$= \tan^{-1}(0.28)$
$= 15.64$

$P = W\tan(\alpha+\phi)$
$= 1000 \tan(35.64)$
$= 716.98N$

c) $\tan\phi = \mu$
$\phi = \tan^{-1}\mu$
$= \tan^{-1}(0.28)$
$= 15.64.$

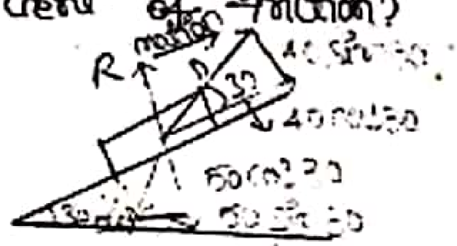
$$P = \frac{W \sin(\alpha+\phi)}{\cos(\theta-\phi)}$$



$$= 1000 \frac{\sin 35.64}{\cos 5.64} = 585.52\,N.$$

(7) A body weighing 50N is just pulled up on inclined plane of 30° by a force of 40N 30° above the plane. Find the co-efficient of friction?

$W = 50N$
$\alpha = 30°$
$P = 40N$
$\theta = 30°$
$\mu = ?$



Resolving forces normal to the plane.

$$R = 50\cos 30 - 40\sin 30 = 43.3 - 20 = 23.3N$$

Resolving forces along

$$40\cos 30 = 50\sin 30 + F$$
$$= 50\sin 30 + \mu \times 23.3$$
$$\mu = 40\cos 30 - 50\sin 30 / 23.3$$
$$= \frac{9.64}{23.3} = 0.418$$

(8) Find the least horizontal force 'p' to start motion of any part of system of 3 blocks resting up on one another as in fig. The weight of the blocks are A=300N, B=1000N, C=2000N, B/w A & B $\mu=0.3$ b/w B & C $\mu=0.2$ & b/w C & the ground $\mu=0.1$?

$W_A = 3000N$

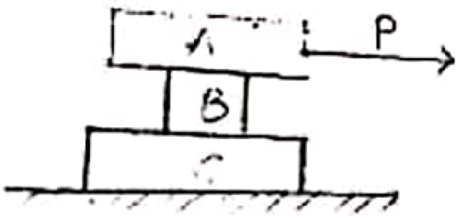
$W_B = 1000N$

$W_C = 2000N$

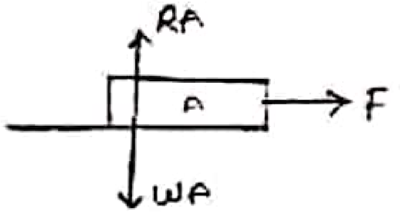
$\mu_{AB} = 0.3$

$\mu_{BC} = 0.2$

$\mu_{Ca} = 0.1$




$P = ?$




Reslovíng forces vertically.

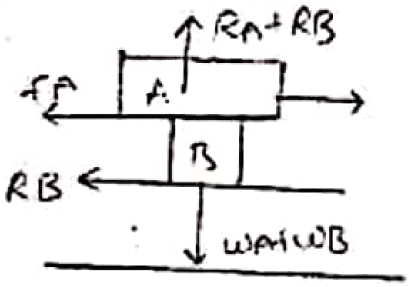
$R_A = W_A = 3000N$

Reslovíng foríes horizontaly.

$P = F_a$

$= \mu_A R_A$

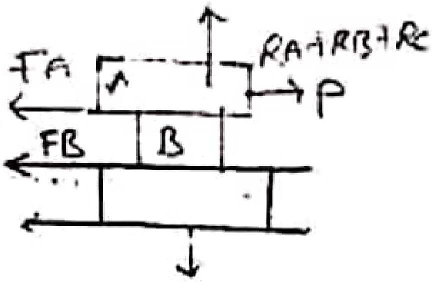
$= 0.3 \times 3000 = 900N$




Reslovíng forces vertically

$R_A + R_B = 3000 + 1000$

$= 4000$

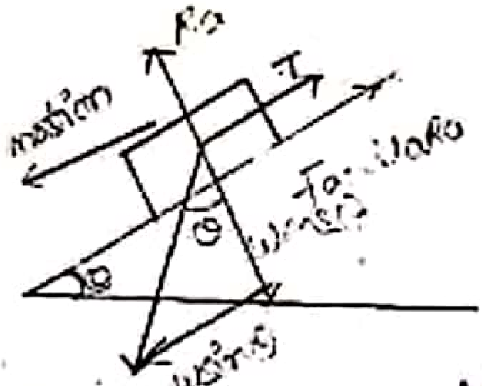



Reslovíng forces horizontaly.

$= F_A + F_B + F_C$

$= 0.3 \times 3000 \times 0.2 \times 1000 + 0.1 \times 2000$

Two equal bodies A and B of weight W each are place-d on rough inclined plane. the bodies are connect-ed by light string. If $\mu_a = \frac{1}{2}$ and $\mu_b = \frac{1}{3}$ show that the bodies both on the point of motion when the plane is inclined at $\tan^{-1}\left(\frac{5}{12}\right)$




Resolving the forces along the plane.

$T + Fa = w\sin\theta$

$T + \mu a Ra - w\sin\theta = 0 \rightarrow ①$

Resolving the force Normal to the plane

$Ra = w\cos\theta$

$Ra - w\cos\theta = 0 \rightarrow ②$

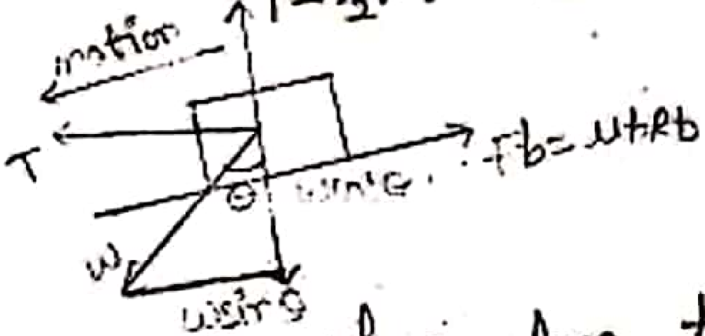
from Eqn ① & ②

$T + \mu a Ra - w\sin\theta = Ra - w\cos\theta$

$T + \mu a Ra - Ra - w\sin\theta + w\cos\theta = 0$

$T + \frac{1}{2} Ra - Ra - w\sin\theta + w\cos\theta = 0$

$T - \frac{1}{2} Ra - w\sin\theta + w\cos\theta = 0 \rightarrow ③$




Resolving forces along the plane.

$T + w\sin\theta = Fb$

$T + w\sin\theta - \mu_b R_b = 0 \longrightarrow (4)$

Resolving Normal to the plane

$R_b = w\cos\theta$

$R_b - w\cos\theta = 0 - (5)$

from (5) & (4)

$T + w\sin\theta - \mu_b R_b = R_b - w\cos\theta$

$T + w\sin\theta - \frac{1}{3}R_b - R_b + w\cos\theta = 0$

$T + w\sin\theta - \frac{4R_b}{3} + w\cos\theta = 0 \longrightarrow (6)$

from (3) & (6)

$T - \frac{1}{2}R_a - w\sin\theta + w\cos\theta = T + w\sin\theta - \frac{4R_b}{3} + w\cos\theta$

$(R_a = R_b)$

$-\frac{1}{2}R_a + \frac{4R_a}{3} - w\sin\theta - w\sin\theta = 0$

$\frac{-3R_a + 8R - 12\, w\sin\theta}{6} = 0$

$5R_a - 12\, w\sin\theta = 0$

$5(w\cos\theta) - 12 w\sin\theta = 0$

$5 w\cos\theta = 12 w\sin\theta$

$\frac{5}{12} = \frac{\sin\theta}{\cos\theta}$

$\frac{5}{12} = \tan\theta$

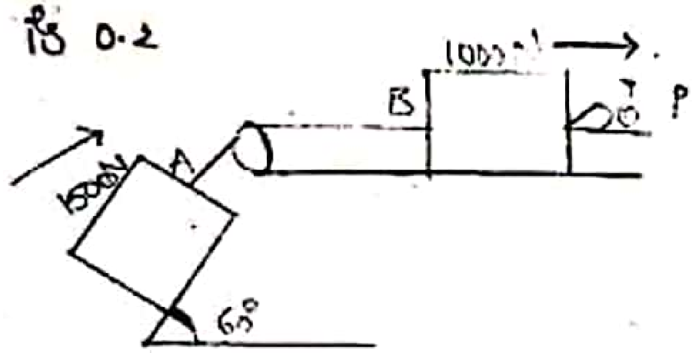
$\theta = \tan^{-1}\left(\frac{5}{12}\right).$

* Referring the figure below determine least value of force p to cause the motion to impend right word Assume co-efficient of friction under the blocks to be 0.2 and the pulley to be frictions?

$\mu$ for blocks is 0.2

$W_a = 1500N$

$W_b = 1000N$



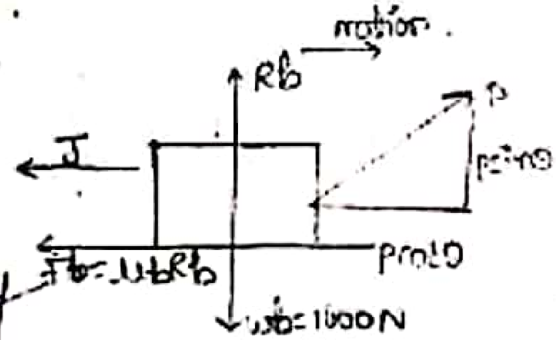
Resolving the forces normal to the plane.

$$R_a = W_a \cos 60$$
$$= 1500 \times 0.5$$
$$R_a = 750N$$

Resolving the forces along the plane.

$$T = F_a + W_a \sin 60$$
$$= \mu_a R_a + 1500 \times 0.866$$
$$= 0.2 \times 750 + 1500 \times 0.866$$
$$= 1449 N$$

Resolving forces vertically.



$$R_b + P\sin\theta = W_b$$
$$R_b = 1000 - P\sin\theta.$$

Resolving forces horizontally.

$$T + F_b = P\cos\theta$$
$$\mu_b R_b = P\cos\theta$$
$$T + 0.2 \times (1000 + P\sin\theta) = P\cos\theta.$$
$$T + 200 - 0.2P\sin\theta = P\cos\theta.$$
$$T + 200 = P\cos\theta + 0.2P\sin\theta.$$
$$T + 200 = P(\cos\theta + 0.2\sin\theta)$$
$$P = \frac{200 + 1449}{(\cos\theta + 0.2\sin\theta)} = \frac{1649}{\cos\theta + 0.2\sin\theta}.$$

To get the values of denomenator is different w.r.t $\theta$

$$\frac{d}{d\theta}(\cos\theta + 0.2\sin\theta) = 0$$

$$-\sin\theta + 0.2\cos\theta = 0$$

$$0.2\cos\theta = \sin\theta$$

$$0.2 = \frac{\sin\theta}{\cos\theta}$$

$$0.2 = \tan\theta$$

$$\theta = \tan^{-1}(0.2)$$

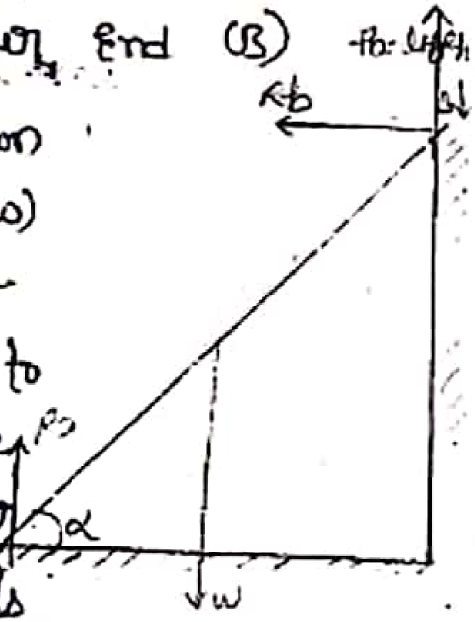
$$\theta = 11.31$$

$$P = \frac{1649}{\cos 11.37 + 0.2\sin 11.31}$$

$$\boxed{P = 1681.88\,N}$$

Ladder friction:-

consider a ladder AB one. End of the ladder (A) is lying on the ground and other end (B) is leaning against wall as shown in figure. Due to the self. weight (w) weight of the man standing on the ladder the End B tends to slipdown words and hence the friction force of b/w the ladder and wall surface acts towords right as the End A tends to slid to left




For Equilibrium of ladder the algebric sum of vertical and horizontal forces be zero and alsi algebric sum of moments about a point must be zero.

A ladder 5m long and 250N weight is placed against a vertical wall in a position where its inclination to the vertical is 30°. A man weighing 800N claim the ladder. At what position will be induced slipping? The coefficient of friction for both contact surfaces of the ladder i.e, with the wall and the floor is 0.2

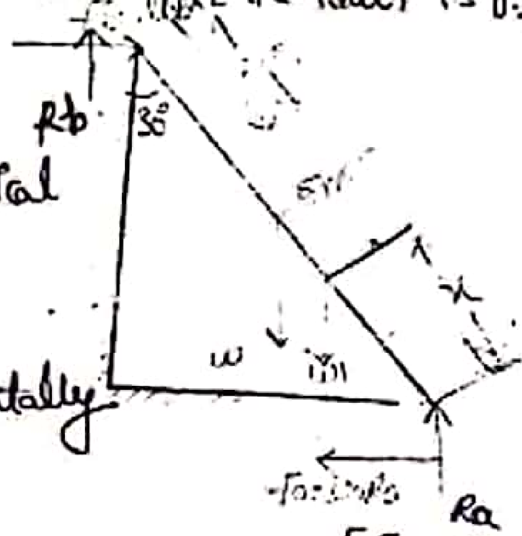
$l = 5m$ Rb

$W = 250N$

30° inclination to the vertical

$W1 = 800N$

$\mu = 0.2$



Resolving the forces horizontally

$Rb = \mu a Ra$

$= 0.2 \times Ra \longrightarrow (1)$

$(\Sigma x = 0)$

Resolving forces vertically.

$Ra + Fb = w + w1$

$Ra + 0.2 Rb = 250 + 800 \longrightarrow (2)$

from (1) & (2)

$Ra + 0.2 \times 0.2 Ra = 1050$

$Ra(1 + 0.04) = 1050.$

$Ra = \frac{1050}{1.04}$

$= 1009.6N$

$Rb = 0.2 \times Ra$

$= 0.2 \times 1009.6$

$= 201.9 N.$

Taking moments about A.

$W \times 2.5 \cos 60 + w \times x \cos 60 = Fb\, 5 \cos 60 + Rb\, 5 \sin 60$

$250 \times 2.5 \times 0.5 + 800x \times 0.5 = 0.2 \times 201.9 \times 0.5 + 201.9 \times 5 \times 0.866$

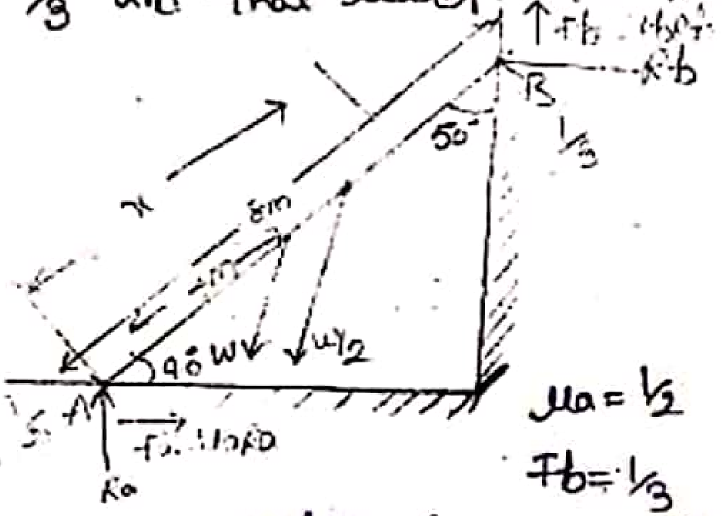
$400x = (0.2 \times 201.9 \times 0.5 + 201.9 \times 0.5 + 201.9 \times 5 \times 0.866 - 250 \times 2.5 \times 0.5$

$400x = 975.25 - 3125$

$x = 1.45 m.$

* V.V.I -

AB in long ladder rest against a vertical wall mak-ing an angle of $50^{\circ}$ with the wall and resting on a floor. If a body whose weight is one half with the ladder climbs it, at whose distance along the ladder will be, when the ladder is about to slip? the coefficient of friction b/w the ladder and wall is $1/3$ and that ladder and floor is $1/2$.



$\mu a = 1/2$

$Fb = 1/3$

Resolving forces vertically

$Ra + Fb = w + w/2$

$Ra + \mu b Rb = w + w/2.$

$$R_a + \frac{1}{3} R_b = \frac{3w}{2} \longrightarrow (1)$$

Resolving forces horizontally

$$R_B = F_a$$

$$R_B = \mu_a R_a$$

$$= \frac{1}{2} R_a \longrightarrow (2)$$

From (1) & (2) eqn.

$$R_a + \frac{1}{3} \times \frac{1}{2} R_a = \frac{3w}{2}$$

$$\frac{6R_a + R_a}{6} = \frac{3w}{2}$$

$$7R_a = \frac{6 \times 3w}{2}$$

$$R_a = \frac{9}{7} \times w$$

$$R_a = \mu_a R_a$$

$$= \frac{1}{2} \times \frac{9}{7} w$$

$$= \frac{9}{14} w$$

Taking moments from 'A'

$$w\,4\cos 40 + \frac{w}{2} x \cos 40$$

$$= R_B\, 8\cos 40 + F_b\, 8 \sin 40$$

$$w(4\cos 40 + \tfrac{1}{2} x \cos 40) = \frac{9}{14} w \times 8\cos 40 + \frac{1}{3} \times \frac{9w}{14} \sin 40$$

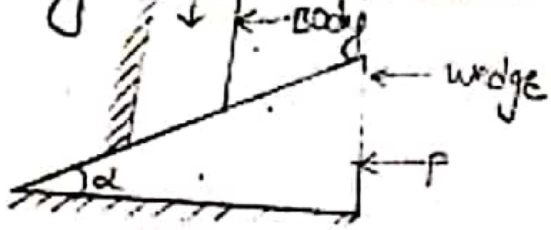
$$\cancel{w}(4\cos 40 + \tfrac{1}{2} \times x \cos 40] = \cancel{w}\left[\frac{9}{14} \times 8\cos 40 + \frac{9}{42} \times \sin 40\right]$$

$$4 \times 0.766 + x \times 0.383 = 5 \times 0.6428 + 1.714 \times 0.642$$

$$x = \frac{[3.3056 + 1.313]}{0.383}$$

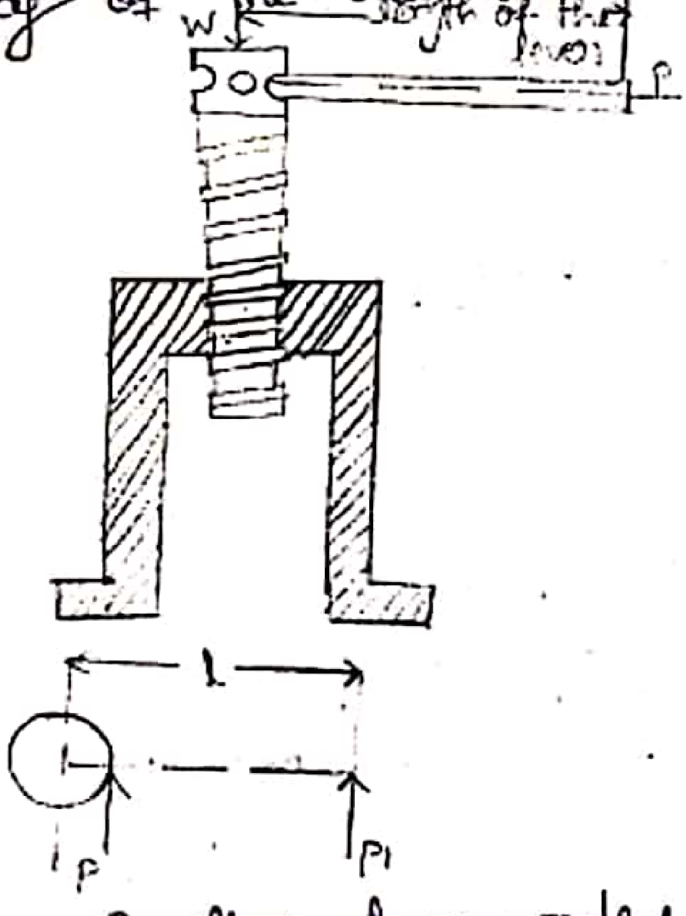
$$x = 8.26$$

**Wedge Friction:-** the wedge is a piece of metal or wood with triangular or trapezoidal cross ~~friction~~ section. It is used splitting devices and lift or adjust the heavy load, with small displacement.



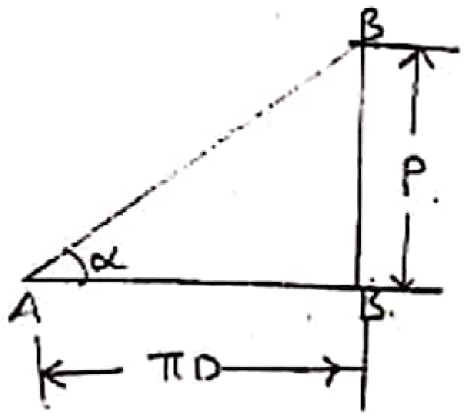

**Screw friction:-**

the general form of simple screw jack is shown in figure It consists of vertical screw and nut. the load rest on the screw head and the nut forms the body of the screw jack.




P = effort ~~of~~ mean radius

P1 = Effort ~~of~~ the end of the lever.




The hight of the plane BC is the distance moved axially in one revolution of the screw in its nut. i.e. pitch (P) The base of the plane (AB) is circumference of the thread at the mean radius i.e, $\pi D$, where D is mean thread diameter.

The angle $\alpha$ of the plane is given by

$$\tan\alpha = P/\pi D$$

for double start thread.

$$\tan\alpha = \frac{2P}{\pi D}$$

for n-start

$$\tan\alpha = \frac{nP}{\pi D}$$

Let

W = axial load on screw

P = Tangental force required at mean radius to turn the screw.

consider the effect P in two cases

(i) Load being raised.

(ii) Load being raised.

Effort required at radius to lift the load. $P = W \tan(\alpha + \phi)$

Torque required to rotate the screw against the load $T = P \times R_m$

$$\boxed{T = W \times \tan(\alpha + \phi) \times R_m}$$

where $R_m$ = mean radius $D/2$

Effort required at the end of the lever.

$P_1 \times l = P \times R_m$

$$\boxed{P_1 = \frac{W \times R_m \times \tan(\alpha + \phi)}{l}}$$

(ii) Load being lowered effort required lowering a load at mean radius.

$$\boxed{P = W \times \tan(\alpha - \phi)} \text{ if } (\alpha > \phi)$$

$$\boxed{= W \times \tan(\phi - \alpha)} \text{ if } (\phi > \alpha)$$

Effort required at the end of the lever

$$\boxed{P_1 = \frac{P \times R_m}{l}}$$

~~Effec~~ Efficiency of screw jack :-

$$\eta = \frac{W \times p}{P \times \pi D}$$

$$\frac{W}{P} = \frac{1}{\tan(\alpha + \phi)}, \quad \frac{p}{\pi D} = \tan\alpha$$

$$\boxed{\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)}}$$

when load falls reverse efficiency used

$$\boxed{\eta = \frac{\tan(\alpha - \phi)}{\tan\alpha}}$$

maximum efficiency

$$\boxed{\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}}$$

Q * I. A screw jack has a square thread of mean diameter 6cm and pitch 0.8cm. The coefficient of friction of the screw thread is 0.09. A load of 14KN is to be lifted through 15cm. Determine the torque required and work done in lifting the load through 15cm. Find the efficiency of the jack also?

$D_m = 6cm$

$R_m = 6/2 = 3cm$

$0.03m$

$P = 0.8cm$

$\mu = 0.09$

$W = 14KN$

~~force~~ $P = w\tan(\alpha+\phi)$

$$\tan\alpha = \frac{P}{\pi D}$$

$$= \frac{0.8}{\pi \times 6} = 0.0424$$

$= \tan^{-1} 0.0424$

$\alpha = 2.43^\circ$

$= w\tan(\alpha+\phi)$

$= 14000\tan(2.43+5.14)$

$= 1860.53$

$T = P \times R_m$

$= 1860.53 \times 0.03$

$= 55.816$ N-m.

$\mu = \tan\phi$

$\phi = \tan^{-1}\mu$

$= \tan^{-1}(0.09)$

$= 5.14$.

work done $= 2\pi N T$

$N = \frac{15}{p} = \frac{15}{0.8} = 18.75$ revolution.

$W = 2 \times \pi \times 18.75 \times 55.816$

$= 6575.6$ N-m

efficiency $(\eta) = \frac{\tan\alpha}{\tan(\alpha+\phi)}$

$= \frac{\tan 2.43}{\tan(2.43+5.14)} \times 100$

$= 31.932\%$

(2) A single threaded screw Jack has a pitch of 12mm and mean radius of 45mm. The coefficient of static friction is 0.15, and kinetic friction is 0.1

(a) Determine the force P applied at the end of the lever. 600mm long which will start lifting a weight of 20,000N.

b) what value of P will keep the screw Jack train

-g.

$R_m = 45$mm

$P = 12$mm

$D_m = 2 \times R_m$

$= 90$mm

$\mu_s = 0.15$

$\mu_k = 0.1$

$P_1 = ?$

$l = 600$mm

$W = 20000$N

$$\tan\alpha = \frac{P}{\pi D} = \frac{12}{\pi \times 90} = 0.092$$

$$\alpha = \tan^{-1} 0.092$$
$$= 2.4$$

$$\mu_s = 0.15$$
$$\phi = \tan^{-1} 0.15$$
$$= 8.53^\circ$$

a) $P = W \tan(\alpha + \phi)$
$$= 20000 \tan(2.4 + 8.53)$$
$$= 3862.25 N$$

$$P_1 = \frac{P \times R_m}{l}$$

$$= \frac{3862.25 \times 45}{600}$$

$$P_1 = 289.66 N$$

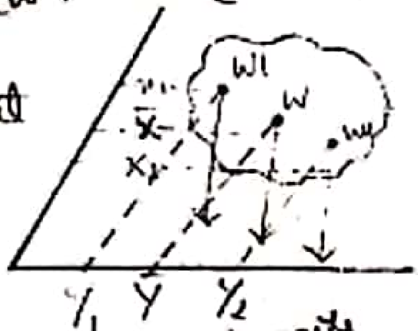
b) $\mu_K = 0.1$
$$\alpha_K = \tan^{-1} \mu_K$$
$$= \tan^{-1} 0.1$$
$$= 5.71$$
$$P = W \tan(\alpha + \phi_K)$$
$$= 20000 \times \tan(2.4 + 5.71)$$
$$= 2849.98 N$$
$$P_1 = \frac{P \times R_m}{l} = \frac{2849.98 \times 45}{600}$$

$$= 213.74 N$$

## Centre of gravity:-

Centre of gravity of a body is defined as the point through which resultant of the gravitational force (weight) acts. consider that the gravitational forces acting on the various particles of the body repres -ent a system of paraller forces The resultant of these parallel forces. is called resultant gravitati -onal force or weight of the body (w) and acts through a point this point is called centre of gravity. The position of centre of gravity can be determined by applying the principle of moments let the weight of indivual particles are $w_1$ and $w_2$ and their coordinates are $(x_1, y_1)$ and $(x_2, y_2)$ respectively. the coordinates of resultant gravi tational force (W) are $(\bar{X}, \bar{Y})$

Taking moments about $\bar{y}$-axis




$W\bar{x} = w_1x_1 + w_2x_2$

the position of C.G from Y- axis

$$\bar{X} = \frac{w_1x_1 + w_2x_2}{W} = \frac{\Sigma wx}{\Sigma W}$$

The position of G.G from x-axis

$$\bar{Y} = \frac{w_1y_1 + w_2y_2}{W} = \frac{\Sigma x}{\Sigma W}$$

Centroid:-

the centre of gravity of thin plate of uniform thickness and homogeneous material is replaced with centroid or centere of area. Centeroid is defi-ned as a point where a whole area of a plane figure assumbed to be concentrated.

$W =$ specific weight x volume

$= w \times v$.

$= (\rho g) \times A \times t$

for homogenous material of uniform thickness, t and $\rho$ are constants.

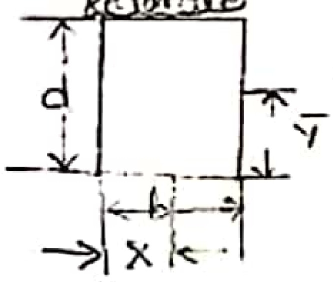
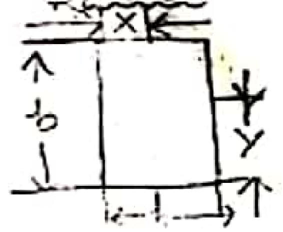
position of centroid from y-axis.

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a} = \frac{\Sigma ax}{a}$$

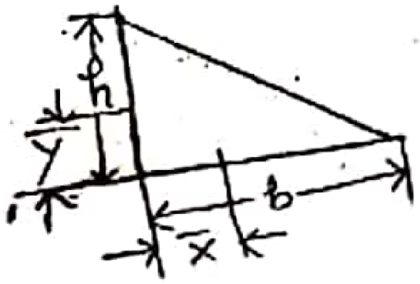
Position of centroid from x-axis

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a} = \frac{\Sigma ay}{\Sigma a}$$

Centroids of plane geometrical shapes:-

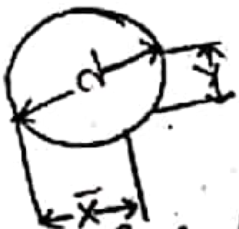
| shape:- | Area | $\bar{X}$ | $\bar{Y}$ |
|---|---|---|---|
| Rectangle  | $b \times d$ | $b/2$ | $d/2$ |
| Square  | $b^2$ | $b/2$ | $b/2$ |

## Triangle:-

| | Area | $\bar{x}$ | $\bar{y}$ |
|---|---|---|---|
|  | $\frac{bh}{2}$ | $\frac{b}{3}$ | $\frac{h}{3}$ |



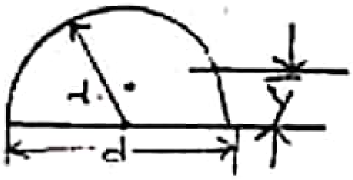
## circle:-





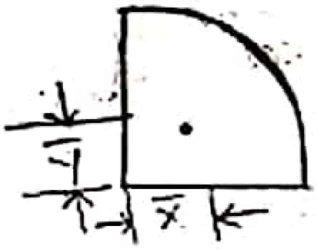
$\pi r^2$ $\quad \frac{d}{2}$ $\quad \frac{d}{2}$

## semicircle:-

| | Area | $\bar{x}$ | $\bar{y}$ |
|---|---|---|---|
|  | $\frac{\pi r^2}{2}$ | $\frac{d}{2} = r$ | $\frac{4r}{3\pi}$ |



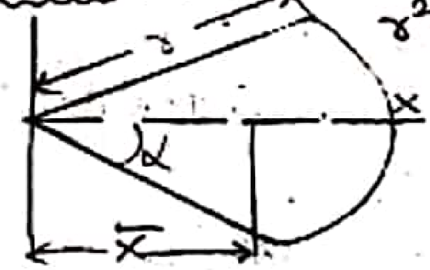
## Quadrant circle:-





$\frac{\pi r^2}{4}$ $\quad \frac{4r}{3\pi}$ $\quad \frac{4r}{3\pi}$

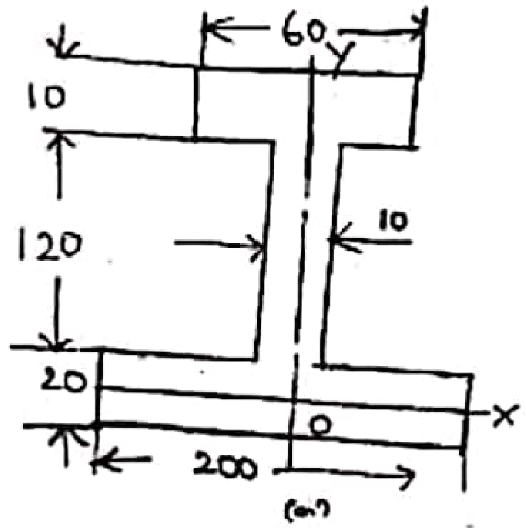
## circular sector:-

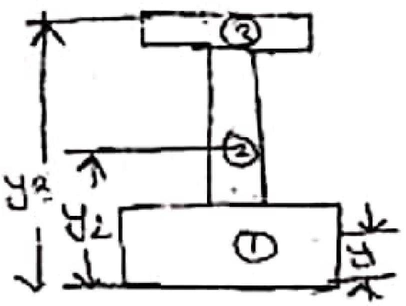




$r^2\alpha$ $\quad \frac{2}{3}\frac{\sin\alpha}{\alpha}$ $\quad 0$

Determine the centroid of I-section of the Dimensions in mm Bottom flange = 200X20 Top flange = 60X10 web = 120X10.




$a_1 = 200 \times 20 = 4000\ mm^2$

$y_1 = d/2 = 20/2 = 10mm$

$a_2 = 120 \times 10 = 1200\ mm^2$

$y_2 = d/2 + 20$

$y_2 = \frac{120}{2} + 20 = 80mm$

$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$

$a_3 = 10 \times 60 = 600\ mm^2$

$= \frac{(4000 \times 10) + (1200 \times 80) + (600 \times 145)}{4000 + 1200 + 600}$

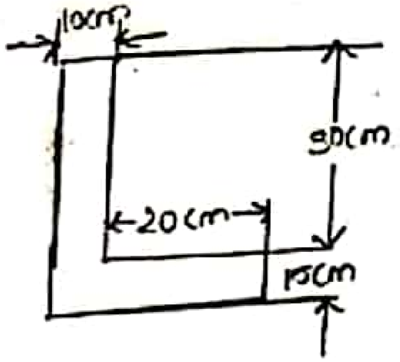
$y_3 = d/2 + 120 + 20$

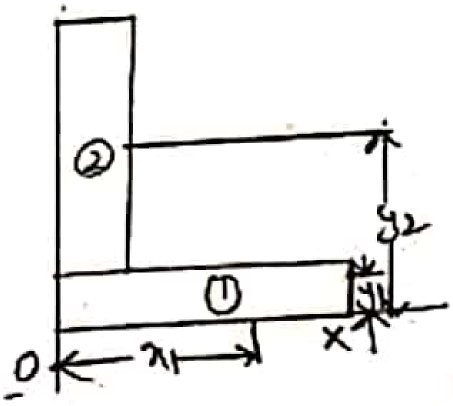
$= 10/2 + 120 + 20$

$= 145 mm.$

$= 38.44 mm$

Find the Centroid of Plane lamina




$a_1 = 15 \times 30 = 450\ cm^2$

$\left(x_1 = \frac{30}{2} = 15,\ y = \frac{15}{2} = 7.5\right)$

$a_2 = 10 \times 30 = 300\ cm^2$

$x_2 = \frac{10}{2} = 5,\ y_2 = \frac{30}{2} + 15 = 30$

Centroid from OY

$$\bar{X} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$= \frac{(450 \times 15) + (300 \times 5)}{(450 + 300)}$$

$$= 11\ cm.$$

Centriod from OX

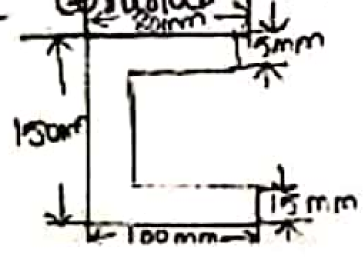
$$\bar{Y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

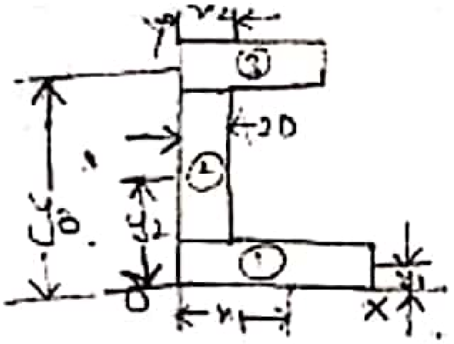
$$= \frac{(450 \times 7.5) + (300 \times 30)}{(450 + 300)}$$

$$= 16.5$$

$$(\bar{X}, \bar{Y}) = (11, 16.5).$$

Find the centroid of plane lamina.






$a_1 = 100 \times 15 = 1500\ mm^2$

$x_1 = \frac{100}{2} = 50,\ y_1 = \frac{15}{2} = 7.5$

$a_2 = 120 \times 20 = 2400\ mm^2$

$x_2 = \frac{20}{2} = 10,\ y_2 = \frac{120}{2} + 15 = 75$

$a_3 = 80 \times 15 = 1200\ mm^2$

$x_3 = \frac{80}{2} = 40,\ y_3 = \frac{15}{2} + 135 = 142.5$

Centroid from oy

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1500 \times 50) + (2400 \times 10) + (1200 \times 40)}{(1500 + 2400 + 1200)}$$

$$= 28.82\ mm.$$

Centroid from ox

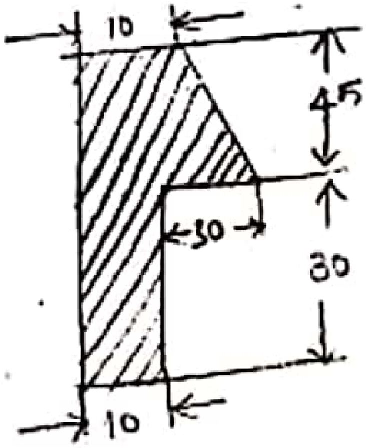
$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

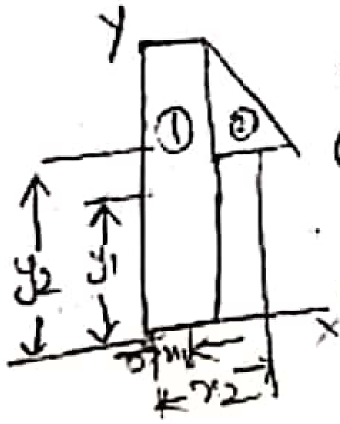
$$= \frac{(1500 \times 7.5) + (2400 \times 75) + (1200 \times 142.5)}{(1500 + 2400 + 1200)}$$

$$= 71.02\ mm.$$

$(\bar{X}, \bar{Y}) = (28.82, 71.02)$

For the shaded area shown in figure determine the coordinates of centroid w.r.t X and y axis. All dimensions are in cm.




$a_1 = 75 \times 10 = 750\,cm^2$

$(x_1 = \frac{10}{2} = 5,\ y = \frac{75}{2} = 37.5)$

$a_2 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 30 \times 45$

$= 675\,cm^2$

$x_2 = \frac{b}{3} + 10 = \frac{30}{3} + 10 = 20$

$y_2 = \frac{d}{3} + 30 =$

$= \frac{45}{3} + 30 = 45.$

centroid from OY

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{(750 \times 5) + (675 \times 20)}{(750 + 675)}$$

$= 12.1\,cm$

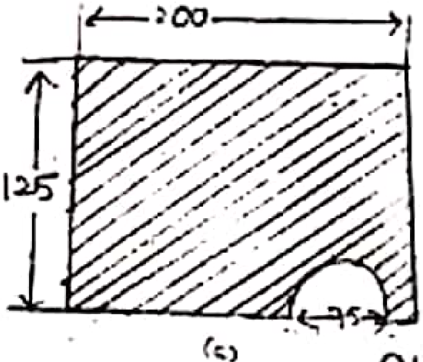
centroid from OX

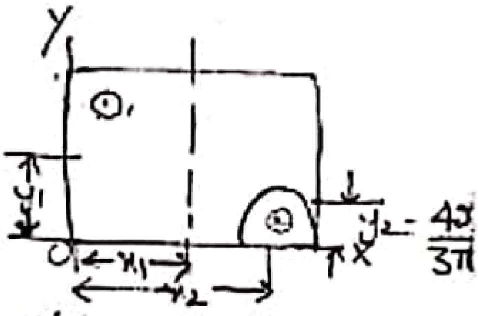
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(750 \times 37.5) + (675 \times 45)}{(750 + 675)} = 41.05\,cm.$$

$(\bar{x}, \bar{y}) = (12.1, 41.05)$

Determine the coordinate of the centroid of the shaded area as shown in figure. If the area removed is semi circular. All dimensions are in mm.




$a_1 = 125 \times 200$

$= 25000\ mm^2$

$x_1 = \frac{200}{2} = 100mm$

$y_1 = \frac{125}{2} = 62.5$

$a_2 = \frac{\pi r^2}{2} = \frac{\pi \times (37.5)^2}{2}$

$= 2208.93\ mm^2$

$x_2 = \frac{75}{2} + 125 = 162.5,$

$y_2 = \frac{4r}{3\pi} = \frac{4 \times 37.5}{3 \times \pi} = 15.4$

centroid from OY

$\bar{x} = \frac{a_1 x_1 \cancel{+} a_2 x_2}{a_1 \cancel{+} a_2}$

$= \frac{(25000 \times 100) - (2208.93 \times 162.5)}{25000 - 2208.93.}$

$= 93.94$ mm.

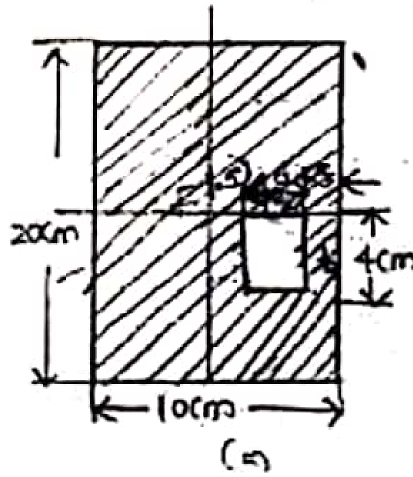
Centroid from OX

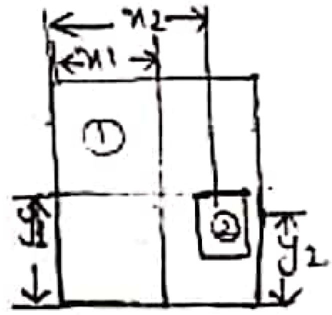
$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$

$= \frac{(25000 \times 62.5) - (2208.93 \times 15.91)}{(25000 - 2208.93}$

$= 67.01 mm.$

From a rectangular lamina shown in figure of dimensions $10 \times 20 cm$, a rectangular hole of $2cm \times 4cm$ is cut. Find the centre of gravity of the remainder.




$a_1 = 20 \times 10$

$= 200 cm^2$

$x_1 = 10/2 = 5cm$

$y_1 = 20/2 = 10 cm$

$a_2 = 4 \times 2$

$= 8 cm^2$

$x_2 = 2/2 + 1.5 + 10/2$

$= 7.5 cm$

$y_2 = 20/2 - 4/2$

$= 10 - 2 = 8cm$

Centroid form OY

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{(200 \times 5) - (8 \times 7.5)}{200 - 8}$$

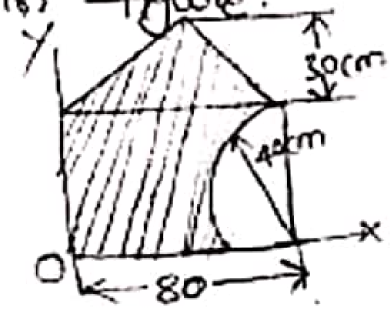
$= 4.89 cm$

Centroid from OX

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$= \frac{(200 \times 10) - (8 \times 8)}{200 - 8} = 10.08 cm$$

Find the centroid of the following shaded area shown in figure.




$a_1 = 80 \times 40$

$= 3200\ cm^2$

$(x_1 = 80/2 = 40,\ y_1 = 40/2 = 20)$

$a_2 = \frac{\pi r^2}{4}$

$= \frac{\pi \times (40)^2}{4}$

$= 1254.64\ cm^2$

$x_2 = 80 - \frac{4r}{3\pi}$

$= 80 - \frac{4 \times 40}{3 \times \pi}$

$= 63.02\ cm^2.$

$y_2 = \frac{4r}{3\pi}.$

$= \frac{4 \times 40}{3 \times \pi}$

$= 16.97\ cm.$

$a_3 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 80 \times 30$

$= 1200\ cm^2$

$x_3 = \frac{b}{2} = \frac{80}{2} = 40$

$y_3 = \frac{h}{3} + 40$

$= 30/3 + 40 = 50$

centroid from OY

$$\bar{X} = \frac{a_1x_1 - a_2x_2 + a_3x_3}{a_1 - a_2 + a_3}$$

$$= \frac{(3200 \times 40) - (1254.64 \times 63.02) + (1200 \times 40)}{(3200 - 1254.64 + 1200)}$$

$$= 30.81\,cm.$$

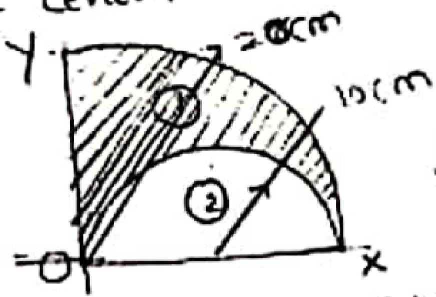
Centroid from OX

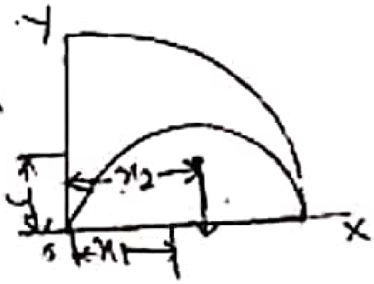
$$\bar{Y} = \frac{a_1y_1 - a_2y_2 + a_3y_3}{a_1 - a_2 + a_3}$$

$$= \frac{(3200 \times 20) - (1254.64 \times 16.97) + (1200 \times 50)}{(3200 - 1254.64 + 1200)}$$

$$= 32.65\,cm$$

Locate the centroid of the shaded area as shown in the figure.




$$a_1 = \frac{\pi r^2}{4}$$

$$= \frac{\pi \times (20)^2}{4}$$

$$= 314.15\,cm^2$$

$$x_1 = \frac{4 \times r}{3\pi}$$

$$= \frac{4 \times 20}{3 \times \pi}$$

$$= 8.48\,mm$$

$$y_1 = \frac{4r}{3\pi}$$

$$y_1 = 8.48\,mm.$$

$$a_2 = \frac{\pi r^2}{2}$$

$$= \frac{\pi \times 10^2}{2}$$

$$= 157.07\,cm^2$$

$$x_2 = \frac{d}{2} = r$$

$$= 10\,cm$$

$$y_2 = \frac{4r}{3\pi}$$

$$= \frac{4 \times 10}{3 \times \pi}$$

$$= 4.24\,cm.$$

centriod from oy

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}$$

$$= \frac{(314.15 \times 8.48) - (157.07 \times 10)}{(314.15 - 157.07)}$$

$$= 6.96 \text{ cm.}$$

Centriod from ox.

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

$$= \frac{(314.15 \times 8.48) - (157.07 \times 4.24)}{(314.15 - 157.07)}$$

$$= 12.71 \text{ Cm}$$

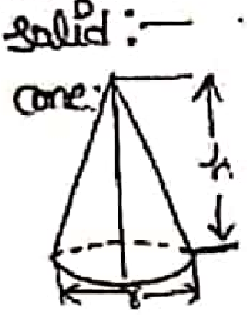
## centre of gravity for solids:—

$$\bar{X} = \frac{v_1x_1 + v_2x_2}{v_1+v_2}$$

$$\bar{Y} = \frac{v_1y_1+v_2y_2}{v_1+v_2}$$

solid :—

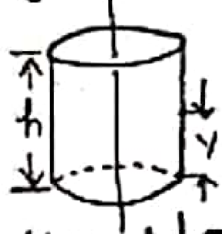
cone:




value $\bar{X}$ $\bar{Y}$

$\frac{\pi r^2 h}{3}$ — $h/4$

cylinder:



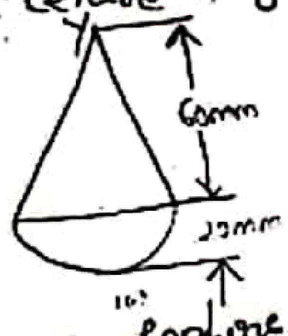

$\pi r^2 h$ — $h/2$

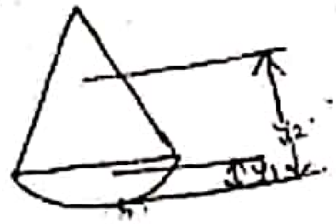
Hemisphere:—


volume $\bar{X}$ $\bar{Y}$.

$\frac{2}{3}\pi r^3$ — $\frac{3}{8}r$.

A solid hemisphere of 20mm radius supports a solid cone of same base and height 60mm in figure. locate the centre of gravity of composite section.




Hemisphere

$V_1 = \frac{2}{3}\pi \times r^3$

$= \frac{2}{3} \times \pi \times (20)^3$

$v_1 = 16755.16\ mm^3$

$y_1 = 20 - \bar{Y}$

$= 20 - \frac{3}{8} \times r$

$= 20 - \frac{3}{8} \times 20$

$y_1 = 12.5mm$

cone:- $V = \frac{\pi r^2}{3} \times h$

$= \frac{\pi \times (20)^2}{3} \times 60$

$v_2 = 25132.74\ mm^3$.

$y_2 = 20 + h/4$

$= 20 + 60/4$

$= 35mm$.

$$\bar{Y} = \frac{v_1 y_1 + v_2 y_2}{}$$

$$= \frac{(16755.16 \times 12.5) + (25132.74 \times 35)}{(16755.16 + 25132.74)}$$

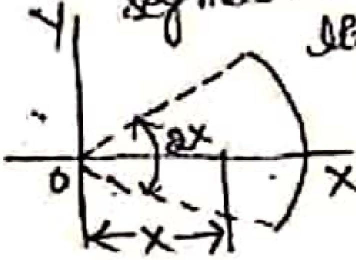
$= 26\,mm.$

**Centroid of lines:—**

$$\bar{X} = \frac{L_1 x_1 + L_2 x_2}{L_1 + L_2}$$

$$= \frac{L_1 y_1 + L_2 y_2}{L_1 + L_2}$$

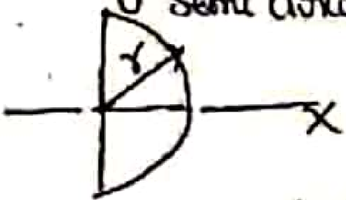
Line segmentation of line | length | $\bar{X}$ | $\bar{Y}$




$2\alpha r$ $\quad \frac{r \sin\alpha}{\alpha}$ $\quad 0$

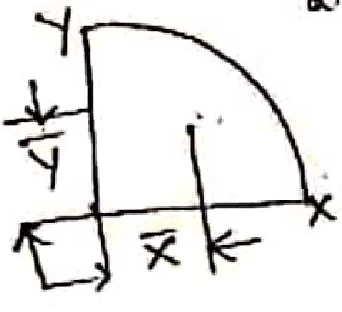
Segmentation of line.

Semi circular arc | length | $\bar{X}$ | $\bar{Y}$



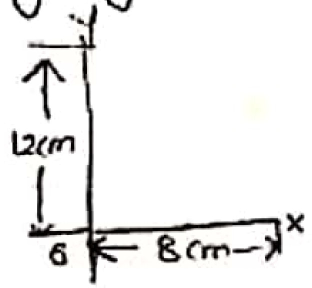

$\pi r$ $\quad \frac{2r}{\pi}$ $\quad 0$

Quadrant circular arc.




$\frac{\pi r}{2}$ $\quad \frac{2r}{2\pi}$ $\quad 2r/\pi$

A wire of length 20cm is bent in the form of L the length of short leg 8cm and long leg 12cm locate the centroid




$l_1 = 8cm$

$x_1 = 8/2 = 4cm$

$y_1 = 0cm$

$l_2 = 12cm$

$x_2 = 0$

$y_2 = 6cm$

centroid from OY

$$\bar{X} = \frac{l_1x_1 + l_2x_2}{l_1+l_2} = \frac{(8\times4)+(12\times0)}{(8+12)}$$

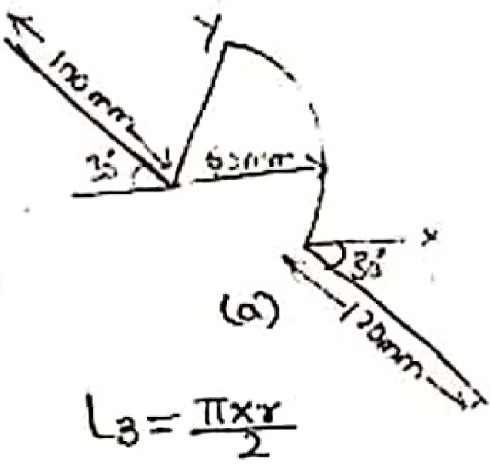
$= 1.6cm.$

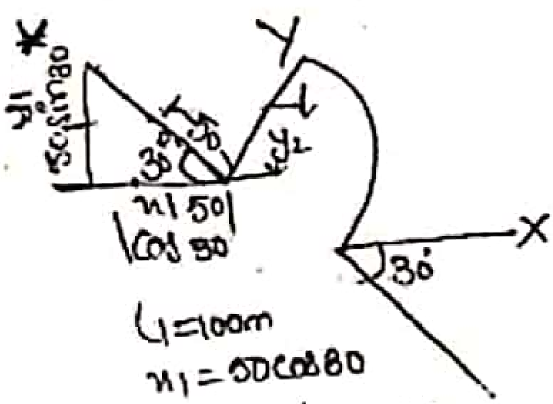
centroid from OX

$$\bar{Y} = \frac{l_1y_1 + l_2y_2}{l_1+l_2}$$

$$= \frac{(8\times0)+(12\times6)}{(8+12)} = 3.6\ cm$$

Locate the centroid of the wire bent as shown in figure




$l_1 = 100m$

$x_1 = 50\cos 30$

$= -43.3mm$

$y_1 = 50\sin 30$

$= 25$

$l_2 = 60mm$ $y_2 = 30mm$

$x_2 = 0mm$

$L_3 = \frac{\pi\times r}{2}$

$= \frac{\pi\times60}{2} = 94.25mm$

$x_3 = \frac{2\times r}{\pi} = 38.18mm$

$y_3 = \frac{2r}{\pi} = \frac{2\times60}{\pi}$

$= 38.18mm.$

$l_4 = 120mm$

$x_4 = 60 \cos 30$

$= 51.96$

$y_4 = 60 \sin 30$

$= 30 mm$

centroid from OY

$$\bar{x} = \frac{l_1x_1 + l_2x_2 + l_3x_3 + l_4x_4}{l_1 + l_2 + l_3 + l_4}$$

$$= \frac{(100 \times -43.5) + (60 \times 0) + (94.24) \times 38.18) + (120 \times 51.96)}{100 + 60 + 94.24 + 120}$$

$= 26.42 mm$

Centroid from OX

$$\bar{y} = \frac{l_1y_1 + l_2y_2 + l_3y_3 + l_4y_4}{l_1 + l_2 + l_3 + l_4}$$

$$= \frac{(100 \times 25) + (60 \times 30) + (94.24)(58.18) + (120 \times 30)}{100 + 60 + 94.24 + 120}$$

$= 30.72 mm.$

Determine the centroid of the parabolic spandrel as shown in figure

Eq$^n$ of parabola

$y = kx^2$

$x = a, \; y = b$

$b = ka^2$

$K = b/a^2$

$y = \frac{b}{a^2} \times x^2$

Area of the strip

$dA = y\,dx$

Total Area $(A) = \int y\,dx$

$\int_0^a \frac{b}{a^2} \times x^2 \times dx$

$\frac{b}{a^2} \times \frac{x^3}{3}$

$\left[\frac{b}{a^2} \times \frac{a^3}{3}\right]$

$\frac{ba}{3}$

centroid of strip oy

from $oy = x$

$\int x\,dA = \int x \cdot y\,dx.$

$\int x \cdot \frac{b}{a^2} \times x^2 dx.$

$\left[\frac{b}{a^2}\; \frac{x^4}{4}\right]_0^a$

$= \frac{ba^2}{4}$

$\bar{x} = \frac{\int_0^a x\,dA}{\int_0^a dA}$

$\frac{\left[\frac{ba^2}{4}\right]}{\left[\frac{ba}{3}\right]} = 3a/4$

centroid of strip

from $ox = y/2$

$\int y/2\,dA$

$= \int y/2 \times y \times dx$

$$\int_0^a \left[\frac{b^2}{a^4} \times x^4\right] \times \frac{dx}{2}$$

$$= \left[\frac{b^2}{a^4} \times \frac{x^5}{5}\right]_0^a \times \frac{1}{2}$$

$$= \left[\frac{b^2 a}{10}\right]$$

$$\bar{Y} = \frac{\int y/2 \, dx}{\int dA} \Rightarrow \frac{\left[\frac{b^2 a}{10}\right]}{\left[\frac{ba}{3}\right]} = \frac{3b}{10}$$

Centroid of spandrel

$$(\bar{X}, \bar{Y}) = \left(\frac{3a}{4}, \frac{3b}{10}\right)$$

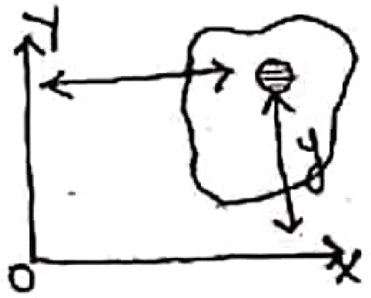
## Unit - 3

## Moment of Inertia

**Moment of inertia:-** moment of inertia is a geometrical characteristics of a cross section of members strength and stiffness of bending member depends on the moment of inertia of its section. The moment of inertia a of its section about axis is defined as the sum of products of element area (dA) and square of its distance from its axis.

Consider a plane fig. of Area A in the xy plane and let (dA) be the element area situated at a distance.

x and y from oy and ox respectively as shown in figure




the m.I of area A about x-axis (ox)

$I_{xx} = \int y^2 dA$.

the m.I of area about Y-axis (oy)

$I_y = \int x^2 dA$.

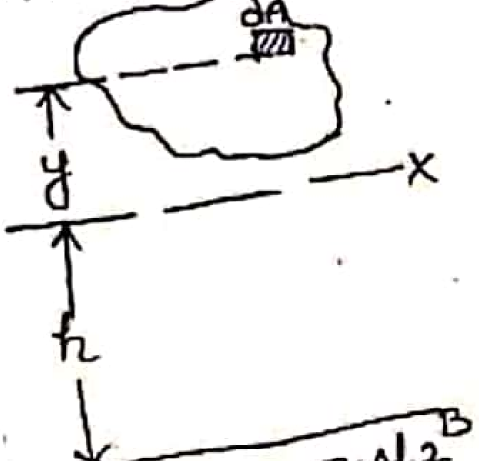
$I = AK^2$

$k = \sqrt{I/A}$ where k is radius of gyration.

Radius of gyration:- consider the entire area concentrated at a point on the lamina. The distance of the point from the given axis of reference is called radius of gyration.

parallel axis theorem:-

It states that the m.I from area from any axis is equal to the m.I about parallel axis passing through centroid plus and area multiplied by the square of the distance between the axis.




A $\quad I_{AB} = I_G + Ah^2$

h = Distance between centroid axis and given axis.

$I_G$ = m.I about centroid axis.

A = Area of given figure

perpendicular axis theorem:—

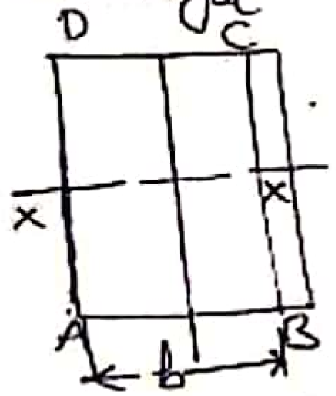
If Ixx and Iyy be the moment of inertia of a lamina about mutually perpendicular to each other. about x-axis and y-axis Then moment of inertia about z-axis normal to the lamina.

$$I_{zz} = I_{xx} + I_{yy}$$

formulas:—

moment of Inertia of plane figures:-

a) Rectangle

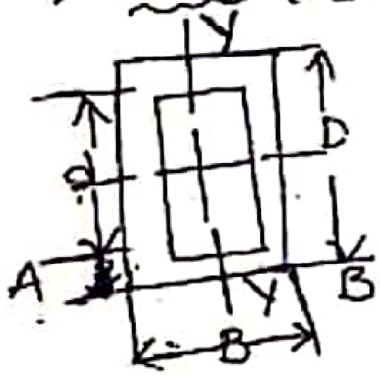



$$I_{xx} = \frac{bd^3}{12}$$

$$I_{yy} = \frac{db^3}{12}$$

$$I_{AB} = \frac{bd^3}{3}$$

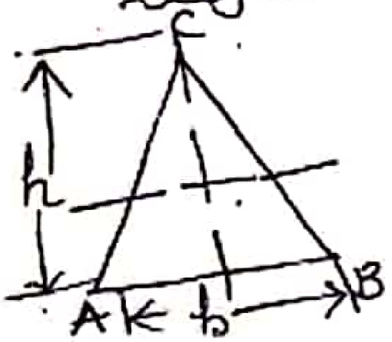
b) Hollow rectangle:—




$$I_{xx} = \frac{BD^3 - bd^3}{12}$$

$$I_{yy} = \frac{DB^3 - db^3}{12}$$

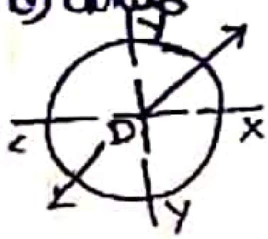
c) triangle




$$I_{xx} = \frac{bh^3}{36}$$

$$I_{AB} = \frac{bh^3}{12}$$

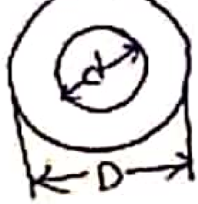
d) circle



$$I_{xx} = I_{yy} = \frac{\pi D^4}{64}$$

$$= \frac{\pi R^4}{4}.$$

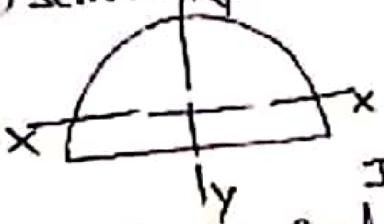
(e) Hollow circle.



$$I_{xx} = I_{yy} = \frac{\pi D^4}{64}$$

$$= \frac{\pi R^4}{4}$$

f) semicircular lamina

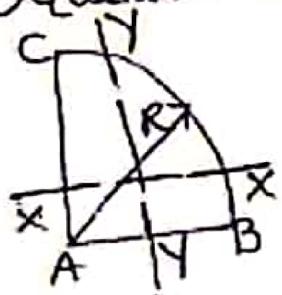


$$I_{AB} = I_{yy}$$

$$= \frac{\pi D^4}{128} = \frac{\pi R^4}{8}$$

$$I_{xx} = 0.11 R^4.$$

g) Quadrant circular lamina



$$I_{AB} = I_{AC}$$

$$= \frac{\pi R^4}{16}$$

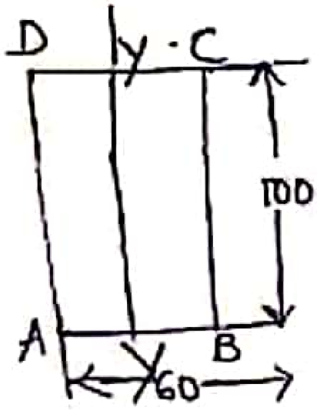
$$I_{xx} = I_{yy}$$

$$= 0.055 R^4$$

1) Find the M.I of a rectangle as shown in figure about

a) centroidal axis ($I_{xx}$ and $I_{yy}$)

b) its base AB.



$$I_{xx} = \frac{bd^3}{12}$$

$$= \frac{60 \times 100^3}{12}$$

$$= 5 \times 10^6$$

$$= 5000000 \text{ mm}^4$$

$$= 5 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{db^3}{12}$$

$$= \frac{100 \times 60^3}{12}$$

$$= 18 \times 10^5 \text{ mm}^4$$

$$I_{AB} = \frac{bd^3}{3}$$

$$= \frac{60 \times 100^3}{3}$$

$$= 2 \times 10^7 \text{ mm}^4.$$

$$I_{AB} = I_G + Ah^2$$

$$= I_{XX} + AX$$

$$= 5 \times 10^6 + (60 \times 100) \times 50^2$$

$$= 2 \times 10^7 \text{ mm}^4.$$

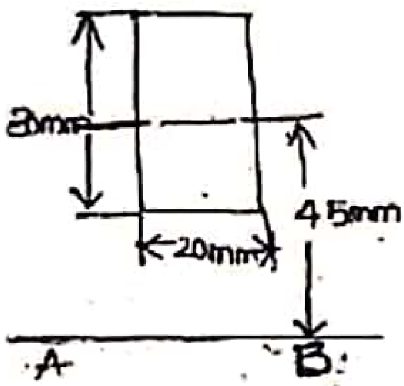
② Find the moment of Inertia of a rectangle 20mm wide, and 30mm deep about a given axis AB, which is the distance of 45mm from its centroid.

$$I_{AB} = I_G + Ah^2$$

$$I_{XX} = \frac{bd^3}{12}$$

$$= \frac{20 \times 30^3}{12}$$

$$= 45000 \text{ mm}^4$$




$$I_{AB} = I_{XX} + A \times h^2$$

$$= 45 \times 10^3 + (20 \times 30) \times 45^2$$

$$= 126 \times 10^4 \text{ mm}^4$$

moment of inertia of composite section:-

the method for finding m.I of a composite section with components having centroidal axis differ from that of the entire section, is outlined below-

(1) Divide the composite section into components.

(2) Locate the Centroid for component section. (97)

(3) Determine the distances $h_1, h_2, h_3 \ldots,$

(4) Compute M.I of each component $I_{G_1}, I_{G_2} \ldots,$

(5) Compute the values $A_1h_1^2, A_2h_2^2$.

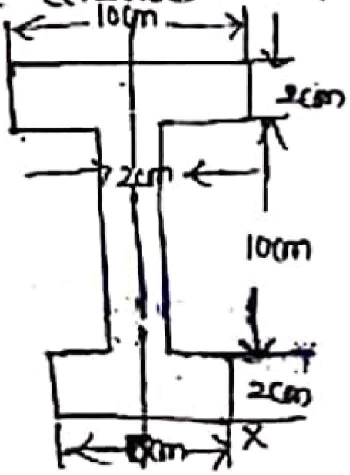
(5) Compute M.I of given section

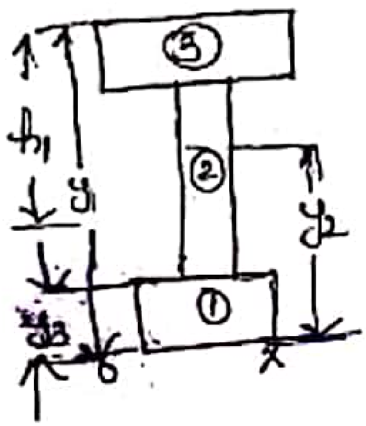
$$I_{XX} = I_{XX_1} + I_{XX_2} + I_{XX_3}$$

$$I_{YY} = I_{YY_1} + I_{YY_2} + I_{YY_3}.$$

Problem:-

Find the M.I (moment of inertia) of the section shown in fig. about the centroidal axis x-x perpendicular to web.




$a_1 = 10 \times 2$
$= 20cm^2$

$y_1 = 2 + 10 + 2/2$
$= 13mm.$

$a_2 = 2 \times 10$
$= 20 cm^2$

$y_2 = 2 + 10/2$
$= 7cm$

$a_3 = \frac{10}{2} \times 2$
$= 10 cm^2$

$y_3 = 2/1 = 1cm$

centroid from OX.

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(20\times 13) + (20\times 7) + (10\times 1)}{(20+20+10)}$$

$$= 8.2\,cm$$

$$I_{xx1} = I_{G1} + a_1 h_1^2$$

$$= \frac{b_1 d_1^3}{12} + a \times h_1^2$$

$$= \frac{10\times 2^3}{12} + (10\times 2) \times (y_1 - \bar{Y})^2$$

$$= \frac{10\times 2^3}{12} + (10\times 2) \times (13 - 8.2)^2$$

$$= 467.46\,cm^4$$

$$I_{xx2} = \frac{b_2 d_2^3}{12} + a_2 h_2^2$$

$$= \frac{2\times 10^3}{12} \times (2\times 10) \times (\bar{Y} - y_2)^2$$

$$= \frac{2\times 10^3}{12} \times 20 \times (1.2)^2$$

$$= 195.47\,cm^4$$

$$I_{xx3} = \frac{b_3 d_3^3}{12} + a_3 h_3^2$$

$$= \frac{5\times 2^3}{12} + (5\times 2) \times (\bar{Y} - y_3)$$

$$= \frac{5\times 2^3}{12} + (10) \times (7.2)^2$$

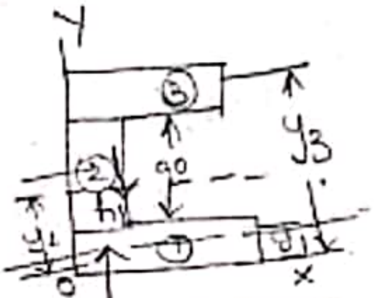
$$= 521.73\,cm^4$$

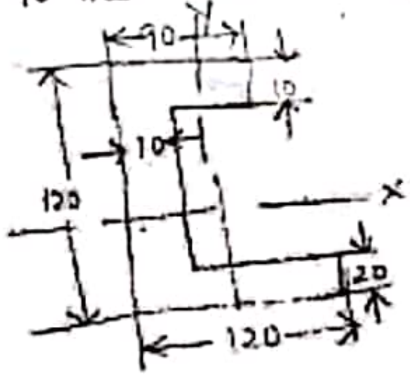
$$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$$

$$= 467.46 + 195.47 + 521.73$$

$$= 1184.67\,cm^4.$$

Find the M.I of section shown in figure about its centroidal axis parallel to the base. All dimensions are in mm





$a_1 = 120 \times 20$

$= 2400 \text{ mm}^2$

$y_1 = 20/2 = 10 \text{ mm}$

$a_2 = 90 \times 10$ $\quad 120 - 10 - 20 = 90$

$= 90 \times 10 = 900 \text{ mm}^2$

$y_2 = \frac{90}{2} + 20 = 65 \text{ mm}$

$a_3 = 90 \times 10$

$= 900 \text{ mm}^2$

$y_3 = \frac{10}{2} + 90 + 20$

$= 115 \text{ mm}$

Centroid from OX

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(2400 \times 10) + (900 \times 65) + (900 \times 115)}{2400 + 900 + 900}$$

$= 44.28 \text{ mm}$

$h_1 = (\bar{Y} - y_1)$

$= (44.28 - 10) = 34.28 \text{ mm}$

$h_2 = (y_2 - \bar{Y})$

$= (65 - 44.28)$

$= 20.72 \text{ mm}$

$h_3 = (y_3 - \bar{Y})$

$= (115 - 44.28)$

$= 70.72\,mm.$

$$I_{xx1} = \frac{b_1 d_1^3}{12} + A_1 \times h_1^2$$

$$= \frac{120 \times 20^3}{12} + (120 \times 20) \times (34.28)^2$$

$$= 29 \times 10^5\ mm^4$$

$$I_{xx2} = \frac{b_2 d_2^3}{12} + A_2 \times h_2^2$$

$$= \frac{(10 \times (90)^3}{12} + (10 \times 90) \times (20.72)^2$$

$$= 99.38 \times 10^4\ mm^4.$$

$$I_{xx3} = \frac{b_3 d_3^3}{12} + A \times h_3^2$$

$$= \frac{90 \times 10^3}{12} + (90 \times 10) \times (70.72)^2$$

$$= 45.086 \times 10^5\ mm^4.$$

$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$

$= 29 \times 10^5 + 99.38 \times 10^4 + 45.086 \times 10^5$

$= 84.02 \times 10^5\ mm^4.$

# kinetics

the study of forces that causes the motion (ex: torque, gravity, friction etc) and classified in to two groups Linear and angular motions.

Kinematics:- The study of describing movements (ex:- are. displacement, time, velocity, etc).

Formula:-

motion under uniform acceleration:-

(i) velocity of a particle $v = u + at$

(ii) Displacement $s = ut + \frac{1}{2}at^2$

(iii) Relation between velocity, acceleration and displacem

$$v^2 - u^2 = 2as$$

where $u$ = initial velocity, m/sec

$v$ = final velocity, m/sec

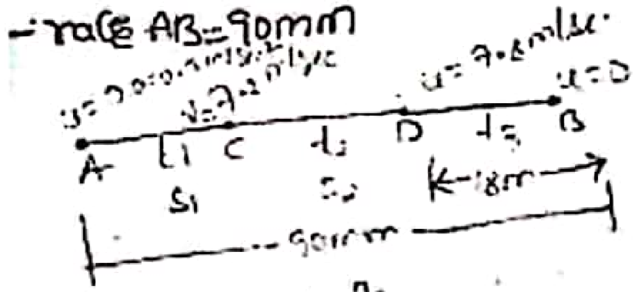
$a$ = acceleration, m/sec$^2$

$s$ = displacement, m.

problem:-

A bus starts from rest at a point 'A' and acceleration at the rate of 0.9 m/sec$^2$ until it reaches a speed of 7.2 m/sec. it is then proceed with the same speed until brakes are applied. It comes to rest at a point B, 18m beyond the point where the brakes are applied

Assuming uniform acceleration determine the time required for the bus travelled from A to B. Distance AB = 90mm



$a_1 = 0.9\ m/sec^2$

$v_1 = 7.2\ m/sec$

$u_1 = 7.2\ m/sec$

$v_1 = u_1 + a_1 t_1$

$$t_1 = \frac{v_1 - u_1}{a_1}$$

$$= \frac{7.2 - 0}{0.9} = 8\ sec$$

$$S_1 = u_1 t_1 + \frac{1}{2} a_1 t_1^2$$

$$= 0 \times 8 + \frac{1}{2} \times 0.9 \times (8)^2$$

$$= 28.8m$$

consider the motion from C to D

$$S_2 = S - (S_1 + S_3)$$

$$= 90 - (28.8 + 18)$$

$$= 43.2m.$$

for uniform velocity

$$S_2 = u_2 \times t_2$$

$$t_2 = \frac{S_2}{u_2}$$

$$= \frac{43.2}{7.2} = 6\ sec.$$

Consider motion from D to B

$$v_3^2 - u_3^2 = 2a_3 s_3$$

$$a_3 = \frac{v_3^2 - u_3^2}{2 \times s_3}$$

$$= \frac{0 - (7.2)^2}{2 \times 18}$$

$$= -1.44 \text{ m/sec}^2$$

$$t_3 = \frac{v_3 - u_3}{a_3}$$

$$= \frac{0 - 7.2}{-1.44}$$

$$= 5 \text{ sec}$$

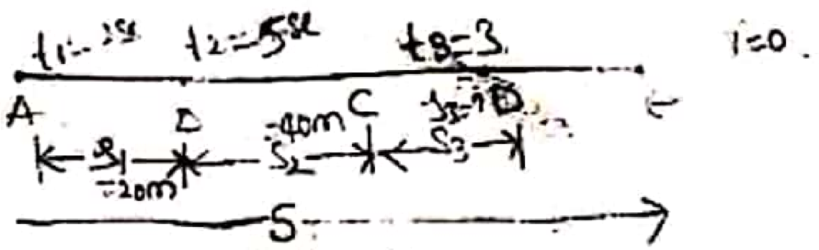
Total time from A to B.

$$T = t_1 + t_2 + t_3$$

$$= 8 + 6 + 5$$

$$T = 19 \text{ sec.}$$

* Q
(2) A particle under a const deceleration is moving in a straight line and cover a distance of 20m in first 2sec. and 40m in next 5 sec. calculate the distance it covers in the subsequent 3 sec. and total distance covered before it comes to re.




$$s = ut + \frac{1}{2} at^2$$

for the motion from A to B

$$20 = u \times 2 + \frac{1}{2} a \times (2)^2$$

$$10 = u + a \quad \text{--- (1)} \qquad 70 = 7u + 7a$$

for the motion from A to C

$$60 = 7 \times u + \frac{1}{2} \times a \times 7^2$$

$$60 = 7u + 24.5a \quad \text{--- (2)}$$

$$7u + 24.5 \times a - 60 = 0$$

$$7u + 7a - 70 = 0$$

$$17.5a + 10 = 0$$

$$a = \frac{-10}{17.5}$$

$$= -0.57 \text{ m/sec}^2$$

$$u = 10 + 0.57$$

$$u = 10.51 \text{ m/sec}$$

for the motion from A to D.

$$(60 + S_3) = 10.51 \times 10 - \frac{1}{2} \times 0.571 \times (10)^2$$

$$S_3 = (77.16 - 60) = 17.16 \text{ m}$$

$$v^2 - u^2 = 2aS$$

$$S = \frac{v^2 - u^2}{2 \times a}$$

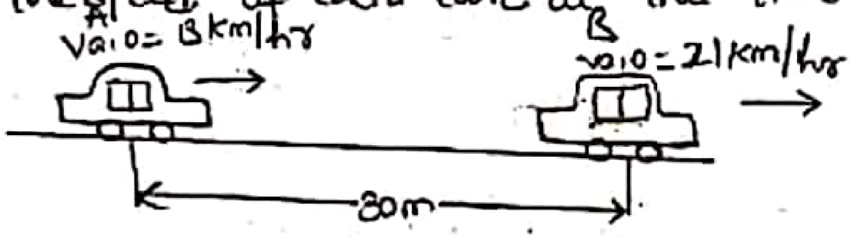
$$= \frac{(0 - 10.51)^2}{2 \times (-0.571)}$$

$$= 97.851 \text{ m.}$$

Two cars A and B are travelling in adjecent highway lanes and at $t=0$ have the position and speed shown in fig. the car A has a constant acceleration 0.8 $m/se^2$ and a car B has a constant decelera-ation of 0.6 $m/se^2$, determine.

a) when and where car A will over take car B car B

b) the speed of each care at that time




sol:- car A overtakes car B after t seconds at a distance x mtores from its start x. meters from its start

$a_a = 0.8 m/sec$ $\quad a_b = -0.6 m/sec^2$

$v_{a,0} = 18 km/hr$ $\quad v_{b,0} = 21 km/hr$

$= 3.61 m/sec$

$v_{a,0} = 18 km/hr$

$= \frac{18 \times 1000}{60 \times 60}$

$= 3.61 m/sec$

$v_{b,0} = 21 m/hr$

$= \frac{21 \times 1000}{3600}$

$= 5.83 m/sec$

consider the motion of car A

$$S_A = u_{a0}\, t + \frac{1}{2} a_a t^2$$

$$= 3.61 \times t + \frac{1}{2} 0.8 \times t^2$$

$$x = 3.61t + 0.4t^2 \quad \text{—— (1)}$$

consider the motion of car B

$$S_{B,t0} = v_B t + \frac{1}{2} a_B t^2$$

$$(x-30) = 5.83 \times t + \frac{1}{2} \times (-0.6) \times t^2$$

$$(x-30) = 5.83t - 0.3t^2 \quad \text{—— (2)}$$

$$(x-30) = 5.83t - 0.3t^2$$

$$(x-30) = 3.61t + 0.4t^2 - 30$$

$$2.22t - 0.7t^2 + 30$$

$$0.7t^2 - 2.22t - 30 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2.22 \pm \sqrt{(-2.22)^2 - 4 \times 0.7 \times -30}}{2 \times 0.7}$$

$$= 8.32$$

$$x = 3.61 \times 8.82 + 0.4 \times (8.82)^2$$

$$= 57.72\,m.$$

velocity of cars after 8.32 sec.

$$v_{a,t} = v_{a,0} + at$$

$$= 3.61 + 0.8 \times (8.82)$$

$$= 10.26\ m/sec$$

$$= 36.936\ m/sec^2$$

velocity of car B

$v_{B,t} = v_{B,0} + at$

$= 5.83 + (-0.6) \times (8.32)$

$= 0.84$ m/sec

$= 3.017$ Km/hr

motion of lift:-

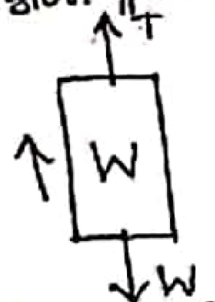
consider a lift moving with uniform acceleration. and carrying some weight consider two cases

a) lift moving upward.

b) lift moving downward.

a) lift moving upward:-

let 'T' be the tension ~~pull~~ in the cable supporting the lift



lift is moving upward.

w be the total weight carried by the lift the net acceleration force.

$(T - w) = m \times a$

$T = w + m \times a$

$= w + \frac{mg \times a}{g}$

$w + w \frac{a}{g}$

$= w\left(1 + \frac{a}{g}\right)$

where

$m$ = mass carried by the lift

$a$ = acceleration of the lift

lift moving downwords:—

As the lift moving downwords the weight is greater than the tension in the cable.

the net accelerating force

$$(w-T) = m \times a$$

$$-T = -w + ma$$

$$T = w - \frac{m \times g \times a}{g}$$

$$= w - w \times a/g$$

$$= w(1 - a/g)$$

(1) A lift has an upward direction of 1.225 m/sec²

a) what pressure will a man weighing 500 N exert on the floor of the lift?

b) what pressure would he exert if the lift had an acceleration of 1.225 m/sec² down words.

c) what upward acceleration would cause his weight to exert a pressure of 600 N on the floor?

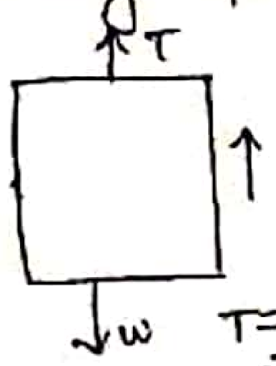
$W = 500 N$

$a = 1.225 \, m/sec^2$

$$m = \frac{w}{g} = \frac{500}{9.81}$$

$$= 50.97 \, kg$$

a) lift is moving upward.

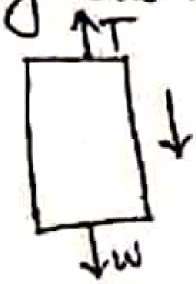



$T = w + ma$

$= 500 + 50.97 \times 1.225$

$= 562.436 N$

b) lift moving downward.




$(w - T) = ma$

$T = w - ma$

$= 500 - 50.97 \times 1.225$

$= 437.56 N$

c) $T = 600 N$

$a =$ upward acceleration

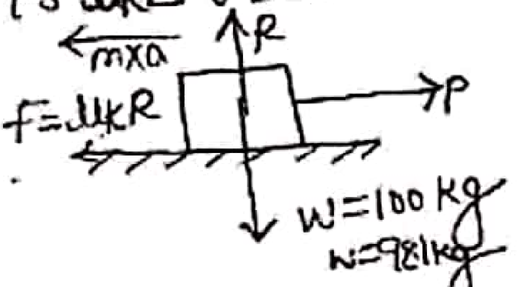
$(T - w) = m \times a$

$a = \frac{(T - w)}{m}$

$= \frac{(600 - 500)}{50.97}$

$= 1.962 \; m/sec^2$

* 100 kg block rest on a horizontal plane, find the magnetude of force required to give the block an acceleration of 2.5 m/sec² to the right. The co-efficent of friction b/w the block and plane is $\mu_k = 0.25$.




$m = 100\,kg$

$w = m \times g$

$= 100 \times 9.81$

$= 981\,N$

$F = \mu_k R$

$= 0.25 \times 981$

$= 245.25\,N$

magnitude of force

$P = F + m \times a$

$= 245.25 + 100 \times 2.5$

$= 495.25$

Kinetics of rigid body :-

(i) Force and translation :- when a rigid body is constrained to move in translation (motion in a straight path) then its angular accelaration is zero.

$F = m \times a$

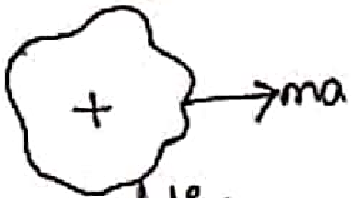
(ii) moment of couple (TORQUE) and rotation :- when a body is constrained to rotate about a fixed axis perpendicular to the reference plane and passing through its centre it is said to be centroidal rotation (rotation about centroidal axis)

$$\text{TORQUE} = I\alpha$$

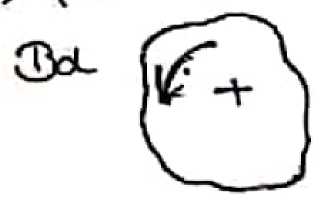
$I$ is mass moment of inertia.

iii) motion of translation and rotation:-

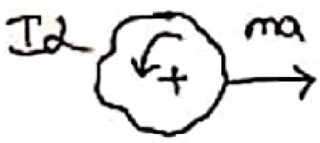
In most cases of kinetics the body moves in a general plane motion which is equal to the sum of transla-tion and centroidal motions. In this case the body simultaneously subjected accelerating force and moment of couple.




a) Translation.




b) Rotation




c) Translation & rotation.

Relation b/w torque and moment of inertia:-

$$T = I\alpha \ \text{N-m}$$

where $I$ = mass m.I kg-m$^2$

$\alpha$ = Angular Acceleration rad/sec$^2$

motion of a body tied to a string passing over a pulley:-

consider a body of mass 'm' connected to a string passing over a pulley as shown in fig.


consider the motion of bo dy

$(w-p)=ma$ —① 

consider the motion of pulley

Torque, $T=P\times r$

$T=I\alpha$

$P\times r=I\alpha$

$P=\frac{I\alpha}{r}$

Linear acceleration

$a=\alpha r$

$\alpha=a/r$

$\therefore P=\frac{I\times a}{r^2}$ —②

from eqn ① & ②

$a=\frac{mg}{\left[\frac{I}{r^2}+m\right]}$ and

$P=\frac{mIg}{I+mr^2}$

If pulley is solid disc of mass m

$I=\frac{mr^2}{2}$

## D'Alembert's principle:-

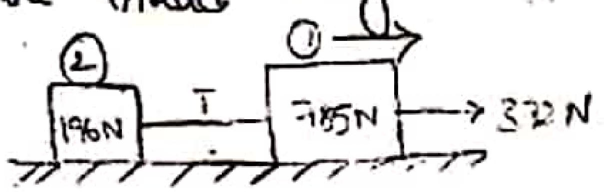
Consider a mass "m" moving with uniform acceleration under the influence of external force, F. Then according to newtons second law of motion

$$F = ma \quad \text{--- (1)}$$

Jean . D 'Alembert introduce the concept of dynamic equilibrium to slove the problem related to motion of bodies D' Alembert principle states that the body will be in dynamic equilibrium under the action of external force (F) and inertia force (ma) . Based on D'Alembert principle eq (1) can be expressed as $F - ma = 0$

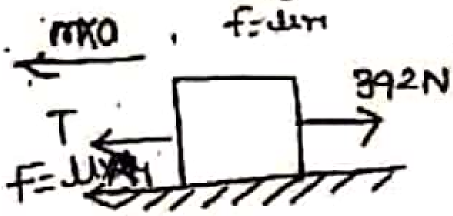
problem:

(1) Two weights 785N and 196N are connected by a thread and moves along a rough horizontal plane under the action of a force 392N applied to the first weight of 785N as shown in the figure. the co-efficient of friction between the sliding surfaces of the weights and the plane is 0.3 Determine the acceleration of weights and tension in the thread using 'D' Alembert principle?



$W_1 = 785N$, $W_2 = 196N$, $P = 392N$, $\mu = 0.3$ $a = ?$

Consider body ①




$392 = T + mxa + F$

$392 - T - mxa + \mu R_1 = 0$

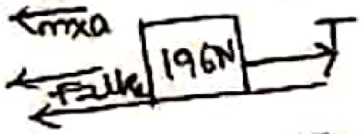
$392 - T - \left(\frac{785}{9.81}\right) xa + 0.3 \times 785 = 0$

$392 - T - \left(\frac{785}{9.81}\right) xa + 0.3 \times 785 = 0$

$627.5 - T - 80.02 \times a = 0$

$T + 80.02 \times a - 627.5 = 0$ — ①

Consider the body ②




$T - mxa - F = 0$

$T - \left(\frac{196}{9.81}\right) xa - 0.3 \times R_2 = 0$

$T - 19.98a - 58.8 = 0$ — ②

$T + 80.02 \times a - 156.5 = 0$

$T - 19.98 \times a - 58.5 = 0$
$(-)$

$100 \times a - 97.7 = 0$

$a = \frac{97.7}{100} = 0.977 \, m/sec^2$

$T + 80.02 \times a - 627.5 = 0$

$T - 19.98 \times a - 58.5 = 0$

$100 \times a - 569 = 0$

$100 \times a = 569$

$a = \frac{569}{100}$

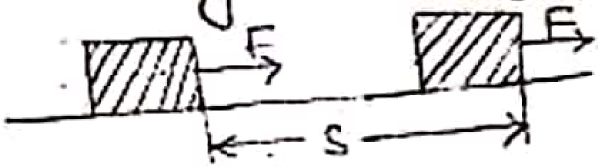
$= 5.69 \, m/sec^2$

$T = 58.5 + 19.98 \times 0.977$

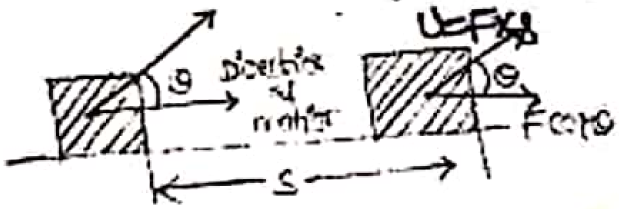
$= 78.82 N$

# Work, power and Energy

Work is done when a force is applied to a body and body moves in the direction of force, it is de-fined as the product of force and displacement of the body in the direction of force.




Work done = Force in the direction of motion × displacement




Work done = Component of force in the direction of motion × displacement

$U = F\cos\theta \times S$

The unit of work is Joule (J)

$1 J = 1 N\text{-}m$

## Power:-

Power is the rate of doing work. The unit of power is watt (W) which is defined as the rate of work equal to 1 J/sec. It is equal to work done by force of one newton in moving through a distance of one metre in one second.

$$\text{power} = \frac{\text{work done}}{\text{time}}$$

The instantaneous power developed by a force F moving at a speed is given by

power = force × speed.

$= F \times v \ \frac{N\text{-}m}{s}, \frac{J}{s}$, watt.

<u>power developed by torque:-</u>

consider a torque, T applied to rotate and angle, θ in time, t then the work done by torque work

done = torque × angle turned

$= T \times \theta$

Rate of doing work

$P = T\theta/t$

But $\theta/t = \omega$

Rate of doing work (power)

$$P = \frac{T \times \theta}{t}$$

$P = T \times \omega$

where T = Torque, N-m

ω = angular speed, rad/sec

If N = No. of revolutions made by an axle per second then

$\omega = 2\pi N$

∴ power developed

$P = 2\pi NT$ watts

Efficiency:- The mechanical efficiency of a machine or engine is defined as ratio of useful work output to the actual work input in a given time

$$\text{Efficiency} = \frac{\text{work output}}{\text{work input}}$$

$$= \frac{\text{power output}}{\text{power input}}.$$

**Energy:-**

The Energy of a body is its capacity to do work there are many different forms of energy such as mechanical energy, heat energy, chemical energy etc.

There two forms of mechanical energys.

1) Kinetic energy
2) potential energy.

**1. kinetic energy:-** The energy possessed by a body by virtual of its motion is called kinetic energy. The kinetic energy of a body of mass 'm' kg moving with velocity of v m/sec. is given as

$$\text{kinetic Energy} = \frac{1}{2} m v^2 \quad N\text{-}m \text{ (or) } J.$$

**2. Work energy equation:-**

work done by a body is equal to change in kinetic energy of the same body

$$\text{force} \times \text{displacement} = \text{change in } K.E$$

ke of body in rotation, work done by torque

= change in K.E of body.

$$W.D = \frac{I \times \omega^2}{2}$$

If $\omega_1$ is initial angular velocity, rad/sec

$\omega_2$ is final angular velocity rad/sec.

W.D by torque = change in K.E

$$W.D = \frac{1}{2} I \left[\omega_2^2 - \omega_1^2\right]$$

(2) <u>potential Energy</u>:- It is the energy possed by a body by virtue of its position.

consider a mass of 'm' kg is raised through a height 'h' above the ground level. Then

$$P.E = W.D = \text{weight} \times \text{height}$$
$$= mgh \quad N\text{-}m \text{ (or) } J.$$


PE=0 $K.E = \frac{1}{2}mv^2$

$$v = \sqrt{2gh}$$

potential energy at B

= kinetic energy at A

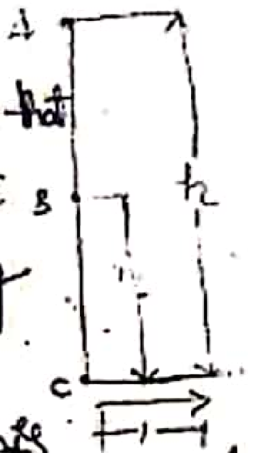
$$mgh = \frac{1}{2}mv^2$$

<u>principle of conservation of energy</u>:-

the law of conservation of energy states that can neither be created nor destroyed, but it can only be transformed from one form to another

law conservation of energy applied to freely falling body:-

from the principle of conservation of energy it is clear that Sum of potential and kinetic Energies of a freely falling body is constant throughout its motion To prove this




consider a body of mass 'm' at height 'h' (i.e, at position 'A' from the ground level 'c')

let 'B' be the another position (mid point b/w C and A) of the body

velocity at 'A' is zero, and let $v_b$ and $v_c$ be the velocity at 'B' and 'c' respectively

$$v_b = \sqrt{2g h/2} = \sqrt{gh}$$

$$v_c = \sqrt{2gh}$$

$$\text{Energy at } A = P.E + K.E$$
$$= mgh + 0$$
$$= mgh$$

$$\text{Energy at } B = P.E + K.E$$
$$= mg h/2 + \tfrac{1}{2} m v_b^2$$
$$= mg h/2 + \tfrac{1}{2} m (\sqrt{gh})^2$$
$$= mg h/2 + \tfrac{1}{2} mgh$$
$$= mgh$$

$$\text{energy at } C = P.E + K.E$$
$$= 0 + \tfrac{1}{2} mv^2 \quad (\because h = 0)$$

$= \frac{1}{2} m (\sqrt{2gh})^2$

$= \frac{1}{2} \times m \times 2gh$

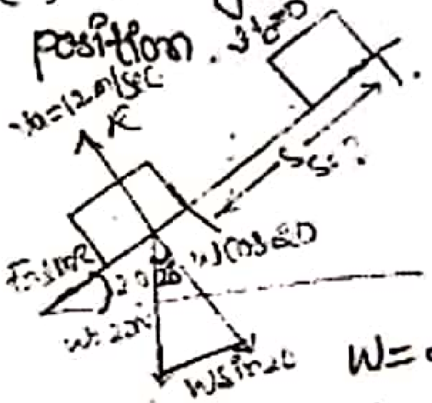
$= mgh$

∴ from the above it is clear that the sum of energy of the body remains const.

(1) A body weighing 20N is projected up a 20° inclined plane with a velocity of 12m/sec co-efficient of friction is 0.15, find

(i) the maximum distance 'S' the body will move up the inclined plane.

(ii) velocity of a body when it returns to original position.




$W = 20N$

$m = \frac{20}{9.81} = 2.039\ kg$

K.E at position on A

$KE = \frac{1}{2} m v_a^2$

$= \frac{1}{2} \times 2.089 \times (12)^2$

$= 146.81\ N-m.$

Total Resistance, R

$R = (F + W\sin 20)$

$R = (\mu R + W\sin 20)$

$= [0.15 \times W\cos 20 + W\sin 20]$

$=\left[0.15\times 20\times \cos 20+20\cdot \sin 20\right]$

$=9.65N$

By principle of work energy

W.D. by resistance = change in K.E

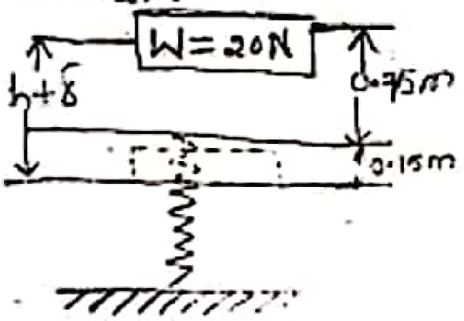
$R\times s=\frac{1}{2}mv_a^2-\frac{1}{2}mv_b^2$

$=\frac{1}{2}\times 2.039\times (12)^2$

$s=\dfrac{0.5\times 2.039\times (12)^2}{9.65}$

$=15.198m.$

A block of weight 20N falls at a distance of 0.75m on top of the spring. Determine the spring stiffness if it is compressed by 150mm to bring the weight momentarily to rest?




$W=20N$

$h=0.75m$

$\delta=150mm$

$=0.15m$

from work. energy principle

$W(h+\delta)=\frac{1}{2}k\delta^2$

$20(0.75+0.15)=\frac{1}{2}\times k[0.15]^2$

$k=\dfrac{2\times 20\times 0.9}{0.0225}$

$=1600\,N/m$

# ENGINEERING MECHANICS

# 1. SIMPLE MACHINES 20 Marks

**Specific Objectives:**

- ✓ Calculate velocity ratio for given machine.
- ✓ Find Efficiency of given machine.

**Contents:**

**1.1 Definitions: ......................................................** **(06 Marks)**

Simple machine, compound machine , load , effort , mechanical advantage , velocity ratio , input of a machine ,output of a machine efficiency of a machine , ideal machine, ideal effort and ideal load, load lost in friction, effort lost in friction.

**1.2 Analysis: ......................................................** **(04 Marks)**

Law of machine, maximum mechanical advantage and maximum efficiency of a machine, reversibility of a machine, condition for reversibility of a machine, self locking machine. Simple numerical problems.

**1.3 Velocity Ratio for Simple Machines: ...................** **(10 Marks)**

Simple axle and wheel, differential axle and wheel, Weston's differential pulley block, single purchase crab, double purchase crab, worm and worm wheel, geared pulley block, screw jack, calculation of mechanical advantage, efficiency, identification of type such as reversible or not etc.

Man invented various types of machines for his easy work. Sometimes, one person cannot do heavy work, but with the help of machine, the same work can be easily done.

To change the tyre of a car, number of person will be required. But with the help of a "Jack", the same work can be done by a single man. Therefore, jack acts as a machine by which the load of a car can be lifted by applying very small force as compared to the load of car.

**Simple Machine or Lifting Machine:**

A machine a device by which heavy load can be lifted by applying less effort as compared to the load.

e.g. Heavy load of car can be lifted with the help of simple screw jack by applying small force.

**Compound Machine:**

Compound machine is a device which may consists of number of simple machines. A compound machine may also be defined as a machine which has multiple mechanisms for the same purpose.

Compound machines do heavy work with less efforts and greater speed.

e.g. In a crane, one mechanism (gears) are used to drive the rope drum and other mechanism (pulleys) are used to lift the load. Thus, a crane consists of two simple machines or mechanisms i.e. gears and pulleys. Hence, it is a compound machine.

**Effort:**

It may be defined as, the force which is applied so as to overcome the resistance or to lift the load.

It is denoted by 'P'.

Magnitude of effort (P) is small as compared to the load (W).

**Load:**

The weight to be lifted or the resistive force to be overcome with the help of a machine is called as load (W).

**Velocity Ratio (V.R.):**

It is defined as the ration of distance traveled by the effort (P) to the distance traveled by the load (W)

$$V.R. = \frac{\text{Distance travelled by effort}}{\text{Distance travelled by load}}$$

Velocity ratio will be always more than one and for a given machine, it remains constant.

**Mechanical Advantage:**

It is defined as the ratio of load to be lifted to the effort applied.

$$M.A. = \frac{\text{Load (W)}}{\text{Effort (P)}} = \frac{W}{P}$$

**Input:**

The amount of work done by the effort is called as input and is equal to the product of effort and distance travelled by it.

Input = P x X, where, P – Effort and X – distance travelled by the effort

**Output:**

The amount of work done by the load is called as output and is equal to the product of load and distance travelled by it.

Output = W x Y where, W – Load and Y – distance travelled by the load



**Efficiency:**

The ratio of output to input is called as efficiency of machine and it is denoted by Greek letter eta (η)

Generally, efficiency is expressed in percentage

$$\%\eta = \frac{\text{Output}}{\text{Input}} \times 100$$

It is always less than 100 because of friction, therefore output < input.

But Output = W.Y and Input = P.X

$$\%\eta = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{W \times Y}{P \times X} \times 100$$

$$\%\eta = \frac{\frac{W}{P}}{\frac{X}{Y}} \times 100 = \frac{M.A.}{V.R.} \times 100 \qquad \left(\text{Since } M.A. = \frac{W}{P} \text{ and } V.R. = \frac{X}{Y}\right)$$

Therefore, efficiency of a machine is also defined as the ratio of mechanical advantage (M.A.) to the velocity ratio (V.R.). It is also expressed in percentage.

$$\% \eta = \frac{\text{M.A.}}{\text{V.R.}} \times 100$$

It is always less than 100 because of friction, therefore M.A. < V.R.

**Actual Machine:**

The machine whose efficiency is always less than 100 % due to frictional resistance offered by the different moving component parts of the machine is called as actual machine.

For such machines, $\eta < 100\ \%$ and hence M.A. < V.R.

**Ideal Machine:**

The machine whose efficiency is 100 % and in which friction is totally absent or zero, is called as ideal machine.

For ideal machines, $\eta = 100\ \%$ and hence M.A. = V.R.

**Ideal Effort (Pi):**

The effort which is required to lift the load when there is no friction is called as an ideal effort (Pi)

Ideal Effort Pi = $\dfrac{W}{\text{V.R.}}$

Where, Pi = Ideal Effort, W = Load to be lifted, V.R. = Velocity Ratio

**Ideal Load (Wi):**

The load which can be lifted by an effort (P), when there is no friction, is called as an ideal load (Wi)

Ideal Load Wi = P x V.R.

Where, P = Effort applied, Wi = Ideal Load, V.R. = Velocity Ratio

**Lever Arm:**

A rigid bar which is provided in machines so as to apply the effort (P) is called as lever arm or handle.

**LAW OF MACHINE:**

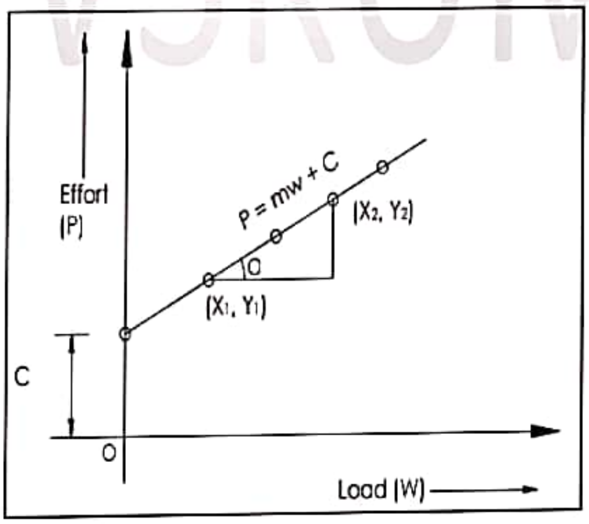
The equation which gives the relation between load lifted and load applied in the form of a slope and intercept of a straight line is called as Law of a machine i.e. **P = mW + C**

Where, P = effort applied, W = load lifted, m = slope of the line and C = y – intercept of the straight line.

To draw the graph of Load (W) V/s Effort (P), effort is applied on a machine and the corresponding values of the loads are noted down. The graph of Load (W) V/s Effort (P) is drawn by taking load (W) on the x-axis and the effort (P) on the y-axis as shown in the figure.



$$m = \tan\theta = \frac{Y_2 - Y_1}{X_2 - X_1}$$




It has been observed that, the graph of load v/s effort is a straight line cuts the Y-axis giving the intercept 'C' which indicates the effort lost on friction, when no load is applied.

Is must be noted that, if the machine is an ideal machine, the straight line of the graph will pass through the origin.

Comparing to the equation of straight line i.e. y = mX + C, we get

**P = mW + C**

Where, P =Effort applied, W = Load applied, m = slope of the line and C = Y-intercept of the line.

***Note:***

*If we know the law of machine i.e. values of 'm' and 'C' are known, then for a given load effort can be found out or for a given effort corresponding value of the given load can be found out. The law of machine also indicates the friction in the machine and maximum M.A.*

**Maximum Mechanical Advantage (Max. M.A.):**

We know that,

$$\mathbf{M.A.} = \frac{\mathbf{W}}{\mathbf{P}}$$

**But,** $\mathbf{P = mW + C}$

$$\therefore \mathbf{M.A.} = \frac{\mathbf{W}}{\mathbf{mW + C}}$$

**Dividing the numerator & Denominator by 'W' we get**

$$\mathbf{M.A.} = \frac{\mathbf{1}}{\mathbf{m} + \frac{\mathbf{C}}{\mathbf{W}}}$$

**In the above equation if 'W' is more, the ratio** $\frac{\mathbf{C}}{\mathbf{W}}$ **will be very small**

$\therefore$ **Neglecting the ratio** $\frac{\mathbf{C}}{\mathbf{W}}$**, M.A. will be maximum**

$$\therefore \mathbf{Max.\ M.A.} = \frac{\mathbf{1}}{\mathbf{m}}$$

**Maximum Efficiency:**

The ratio of maximum M.A. to the V.R. is called as maximum efficiency.

It is also expressed in percentage as

$$\therefore \%\ \text{Maximum}\ \eta = \frac{\text{Max M.A.}}{\text{V.R.}} \times 100 = \frac{1}{m} \times \frac{1}{\text{V.R.}} \times 100 \qquad \left(\text{Since Max M.A.} = \frac{1}{m}\right)$$

**Reversible Machine:**

When a machine is capable of doing some work in the reverse direction even on removal of effort, it is called as reversible machine.

**Condition for Reversible Machine:**

The efficiency of the machine should be more than 50%.

**Irreversible Machine / Non-reversible Machine / Self Locking Machine:**

When a machine is not capable of doing some work in the reverse direction even on removal of effort, it is called as irreversible machine or non-reversible machine or self locking machine.

**Condition for Irreversible Machine:**

The efficiency of the machine should be less than 50%.

**Friction in Machines in terms of Effort and Load:**

In any machine, there are number of parts which are in contact with each other in their relative motion. Hence, there is always a frictional resistance and due to which the machine is unable to produce 100 % efficiency.

Let, P = Actual Effort, $P_f$ = Effort Lost in friction, Pi = Ideal Effort

$\therefore$ **Effort Lost in friction ($P_f$) = Actual Effort (P) – Ideal Effort (Pi)**

$$\therefore\ P_f = P - P_i = P - \frac{W}{V.R.} \qquad \left(\text{Since } P_i = \frac{W}{V.R.}\right)$$

Let, W = Actual load lifted, $W_f$ = Load Lost in friction, Wi = Ideal Load

$\therefore$ **Load Lost in friction ($W_f$) = Ideal Load (Wi) – Actual load lifted (W)**

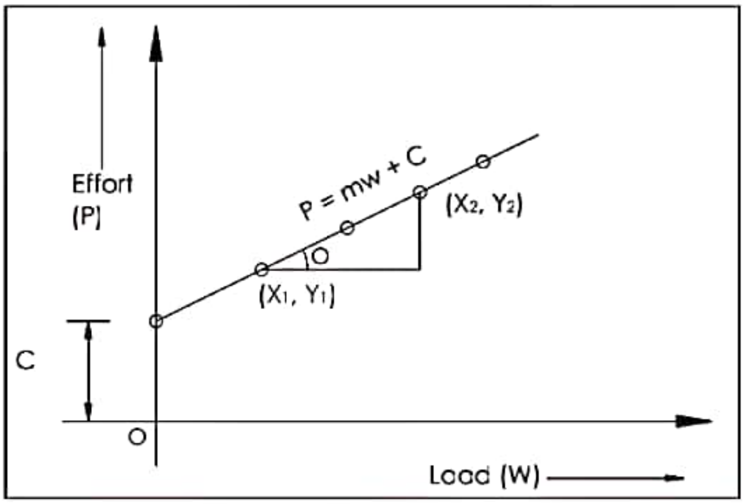
$$\therefore\ W_f = W_i - W = (P \times V.R.) - W \qquad (\text{Since } W_i = P \times V.R.)$$

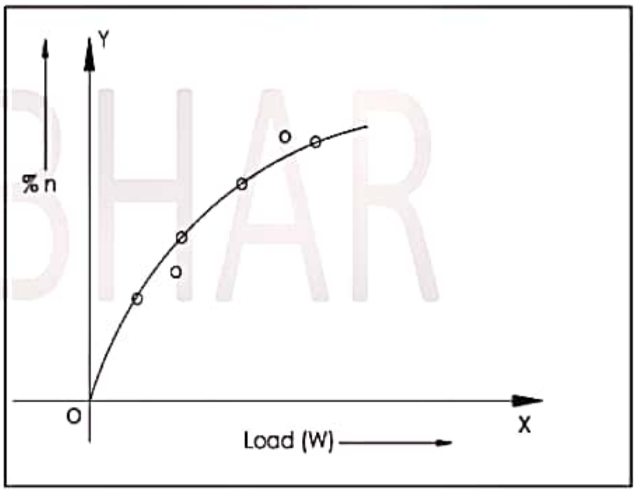
## GRAPHS:

### 1. Load v/s Effort:

The graph of load against effort is a straight line, cuts the y-axis giving the intercept 'C' which represents the effort lost in friction at zero load.

$$m = \tan\theta = \frac{Y_2 - Y_1}{X_2 - X_1}$$

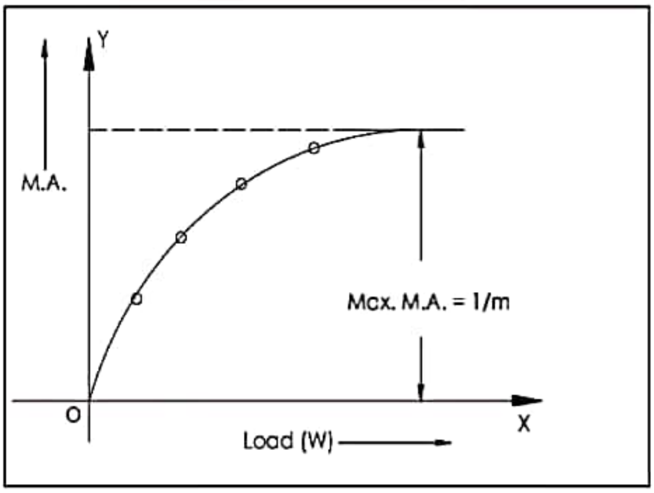



### 2. Load v/s Percentage Efficiency (%η):

The graph of load v/s % efficiency is a curve as shown in the above figure. As load increases, percentage efficiency also increases and therefore gives rise to a smooth curve gradually increasing and becomes more or less parallel to x-axis.

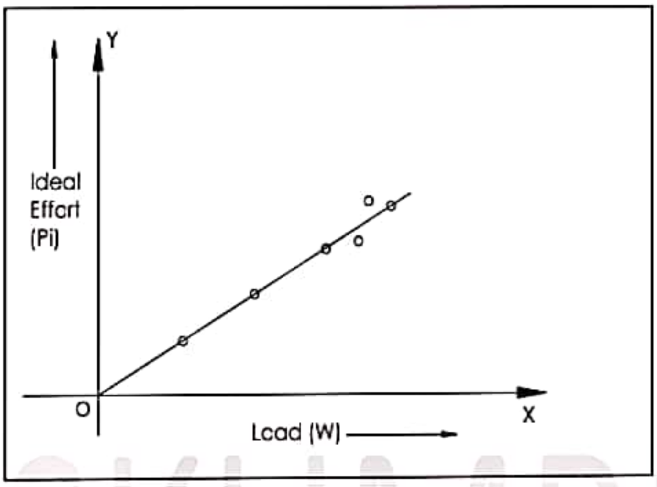
### 3. Load v/s Mechanical Advantage:




The graph of load v/s Mechanical Advantage is a curve as shown in the above figure. As load increases, mechanical advantage also increases and therefore gives rise to a smooth gradually increasing curve.

**4. Load v/s Ideal Effort (Pi):**

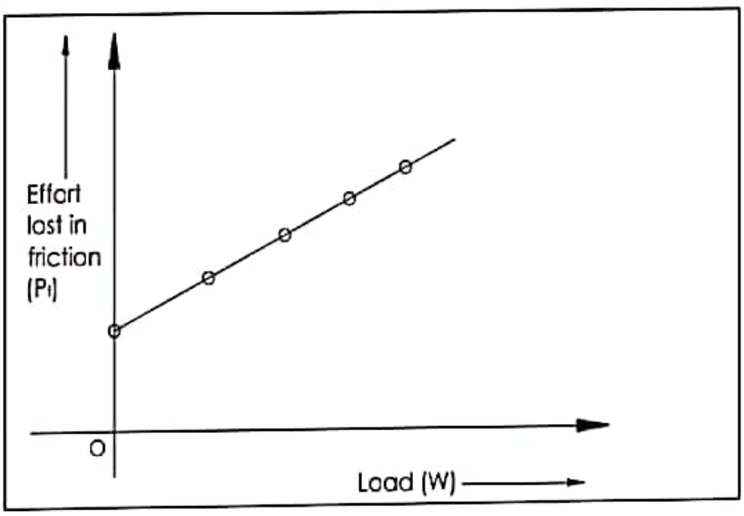
The graph of load v/s ideal effort is a straight line passing through origin as shown in the above figure.






**5. Load v/s Effort lost in friction ($P_f$):**

The graph of load against effort lost in friction is a straight line as shown in the figure.



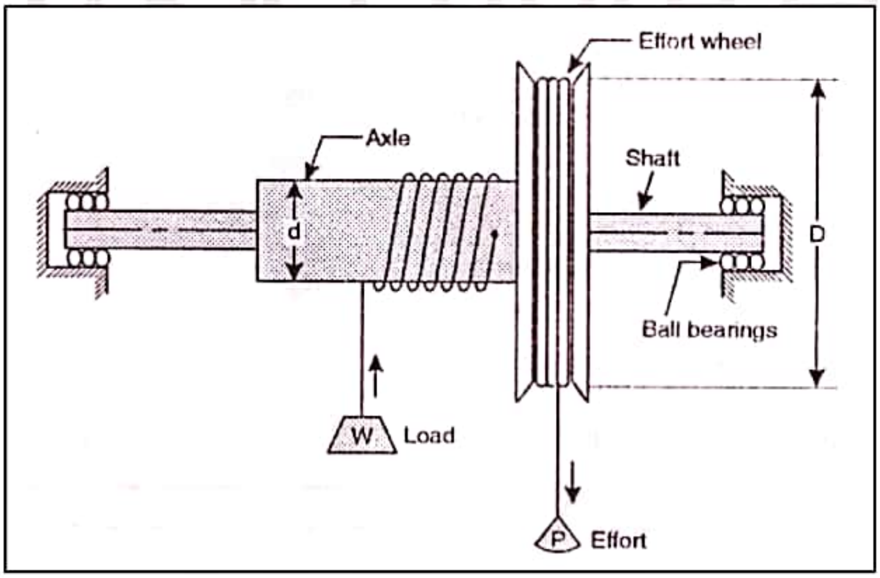
**STUDY OF SOME SIMPLE MACHNES:**

We know that, velocity ratio of a machine is the ratio of distance travelled by the effort to the distance travelled by the load. It is observed that, the distance travelled by the effort is greater than the distance moved by the load. Experimentally, it has been found that, the velocity ratio remains constant for all loads. The velocity ratio changes from machine to machine but remains constant for a given machine.

**1. Simple Wheel and Axle:**

In simple wheel and axle, effort wheel and axle are rigidly connected to each other and mounted on a shaft. A string is wound round the axle so as to lift the load (W) another string is wound round the **effort wheel** in opposite direction so as to apply the effort (P) as shown in the figure.

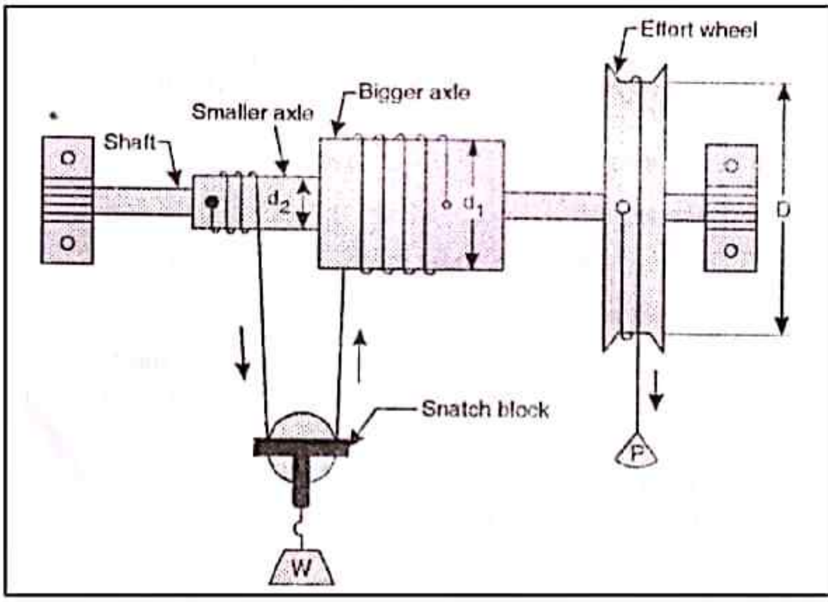
Let, W = Load lifted, P = Effort Applied D = Diameter of the effort wheel d = diameter of the load axle.




When, the effort wheel completes one revolution, the effort moves through a distance equal to the circumference of the effort wheel ($\pi D$) and simultaneously the load moves up through a distance equal to the circumference of the load axle ($\pi d$)

$$\therefore \text{V.R.} = \frac{\text{Distance travelled by the Effort}}{\text{Distance travelled by the Load}} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

**2. Differential Axle and Wheel:**




Differential axle and wheel is a further modification and improvement over the simple axle and wheel.

It consists of a **load axle** made up of **bigger axle** of diameter $d_1$ and **smaller axle** of diameter $d_2$ rigidly connected to each other and an effort wheel of diameter 'D'. **Since, the load axle is made up of two axles of different diameters; it is called as a differential axle.** Differential axle and effort wheel are mounted on the same shaft which is supported on ball bearings as shown in the figure.

A string is wound round the effort wheel so as to apply the effort 'P'. Another string is wound round the bigger axle further passing over the pulley carrying the load 'W' attached to the snatch block. The same string is further wound round the smaller axle in the opposite direction to that of bigger axle. The winding of string on effort wheel and smaller axle is done in the same direction; the string unwinds from the effort wheel & smaller axle and winds over the bigger axle simultaneously, when the effort 'P' is applied.

When effort wheel complete one revolution, the differential axle also completes one revolution.

Distance travelled by the effort = $\pi D$

Length of the string **wound** over the bigger axle = $\pi d_1$

Length of the string **unwound** over the smaller axle = $\pi d_2$

Total winding over the bigger axle = $\pi d_1 - \pi d_2 = \pi(d_1 - d_2)$

But, the load 'W' is lifted through half of the total winding because snatch block with a movable pulley supports the load 'W'

$$\therefore \text{Distance travelled by the load} = \frac{1}{2}(\text{total winding}) = \frac{1}{2}\pi(d_1 - d_2) = \frac{\pi}{2}(d_1 - d_2)$$

We know that,

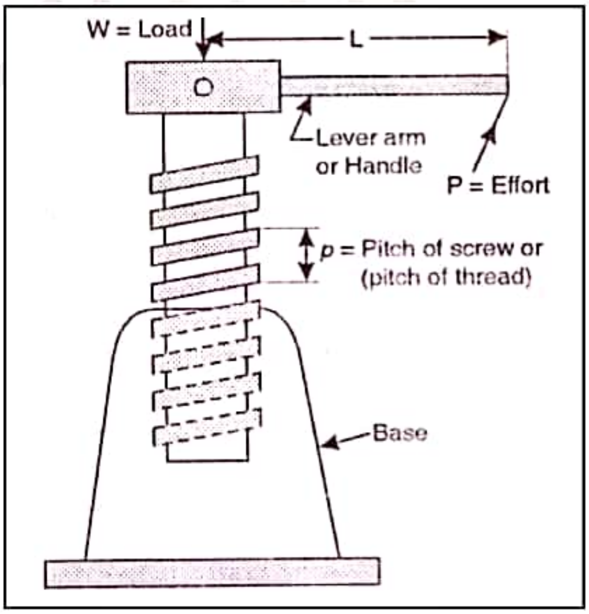
$$\text{V.R.} = \frac{\text{Distance travelled by the effort}}{\text{Distance travelled by the load}} = \frac{\pi D}{\frac{\pi}{2}(d_1 - d_2)}$$

$$= \frac{2D}{d_1 - d_2}$$

Where, D = Diameter of effort wheel, $d_1$ = diameter of bigger axle, $d_2$ = diameter of smaller axle.

If radius of effort wheel, bigger axle and smaller axle are given then,






$$\text{V.R.} = \frac{2R}{R_1 - R_2}$$

Where, R = Diameter of effort wheel, $R_1$ = diameter of bigger axle, $R_2$ = diameter of smaller axle.

### 3. A Simple Screw Jack:

A screw jack is commonly used for lifting and supporting the heavy load. A very small effort can be applied at the end of the lever or handle or tommy bar for lifting the heavy loads. This effort is very small as compared to the load to be lifted. As jack has a simple mechanism, it is most commonly used in repair work of vehicles.

When the effort is applied to the handle or lever arm to complete one revolution then load is lifted through one pitch of the screw (p), therefore the distance moved by the load is equal to the pitch of the screw and the distance moved by the effort is equal to $2\pi L$

Let, L = length of the handle or lever arm and p = pitch of the thread or screw, then

$$V.R. = \frac{\text{Distance travelled by the effort}}{\text{Distance travelled by the load}} = \frac{2\pi L}{p}$$



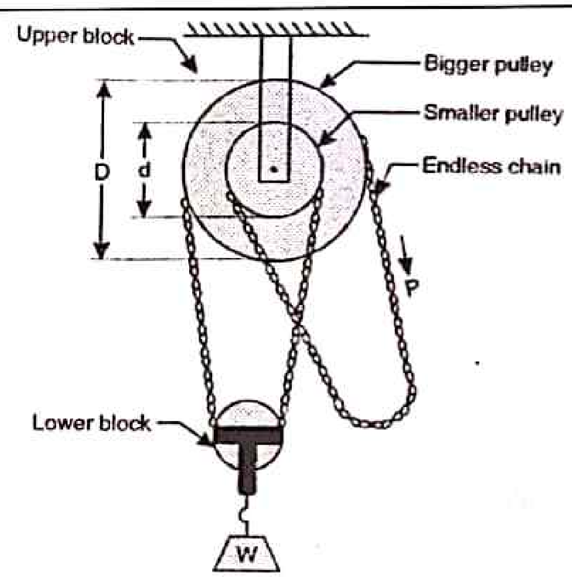
If the effort wheel is used at the place of handle or lever arm for applying the effort, then

$$V.R. = \frac{\text{Distance travelled by the effort}}{\text{Distance travelled by the load}} = \frac{\pi D}{p}$$

Where, D = diameter of the effort wheel and p = pitch of thread or screw.

### 4. Weston's Differential Pulley Block:

It consists of **upper** and **lower block**. Upper block is having bigger pulley and smaller pulley of different diameter mounted on same common axle and that of lower block is having a single pulley. The weight 'W' is attached to the lower block. Upper block is fixed and lower block is movable.



An endless chain passes over the bigger and pulley and single pulley in lower block as shown in the figure.

When there is one complete revolution of the bigger pulley, distance moved by the effort is equal to the circumference of the bigger pulley ($\pi D$). When the bigger pulley completes one revolution, the smaller pulley also completes one revolution because both pulleys are fixed on the same axle. Therefore, for one complete revolution, the length of the chain wound round the bigger pulley is equal to the circumference of the bigger pulley ($\pi D$) and at the same time, the length of the chain unwound from the smaller pulley is equal to the circumference of the smaller pulley ($\pi d$).

$\therefore$ Net winding = $\pi D - \pi d$

As the load (W) attached to the lower block, is equally distributed between the two parts of the chain, distance moved by load is equal to the half of the net winding.

$\therefore$ Distance moved by the load = $\frac{1}{2}(\pi D - \pi d)$

$$\text{V.R.} = \frac{\text{Distance travelled by the effort}}{\text{Distance travelled by the load}} = \frac{\pi D}{\frac{1}{2}(\pi D - \pi d)}$$

$$\therefore \text{V.R.} = \frac{2D}{D - d}$$

Where, D = diameter of bigger or upper pulley, d = diameter of smaller or lower pulley

If radii of bigger and smaller pulley are given, then

$$\therefore \text{V.R.} = \frac{2R}{R - r}$$

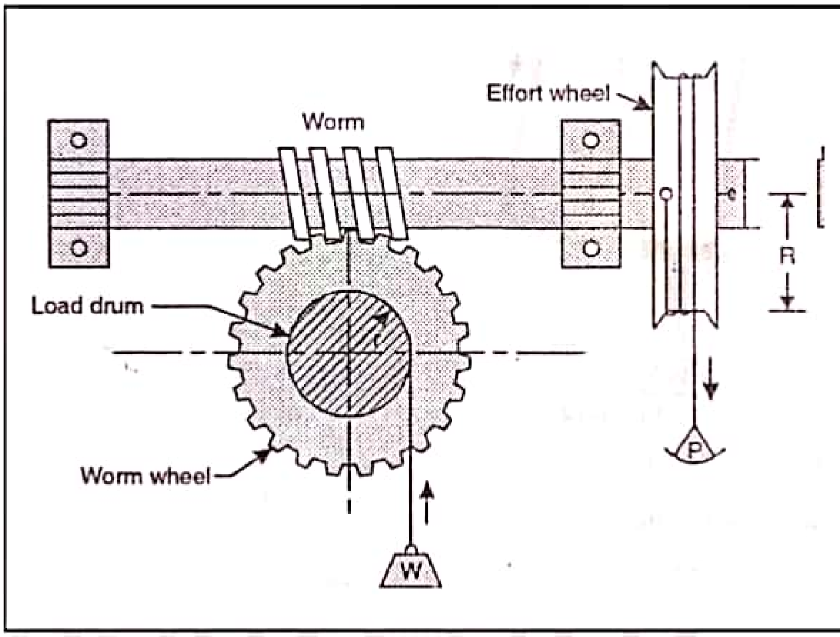
Where, R = radius of bigger or upper pulley, r = radius of smaller or lower pulley

If number of teeth or cogs of the bigger and smaller pulley are given, then

$$\therefore \text{V.R.} = \frac{2T_1}{T_1 - T_2}$$

Where, $T_1$ = number of teeth or cog on bigger or upper pulley, $T_2$ = number of teeth or cog on smaller or lower pulley

**5. Worm and Worm Wheel:**




This machine is made of toothed wheel known as **worm wheel** and a square threaded screw known as **worm**. Worm and worm wheel is geared with each other. The effort wheel is attached to the worm. A load drum is centrally fixed to the worm wheel as shown in the figure. The effort can be either by wheel or handle. The worm may be single threaded or multi-threaded.

Let, w = load lifted, T = number of teeth on the worm wheel, P = effort applied, r = radius of load drum, R = radius of effort wheel.

Consider the worm is single threaded.

For one complete revolution of effort wheel, distance moved by the effort P = 2 π R and load drum performs (1/T) revolution.

$\therefore$ Distance moved by the load = $\frac{2 \pi r}{T}$

We know that,

$$V.R. = \frac{\text{Distance travelled by the effort}}{\text{Distance travelled by the load}} = \frac{2\pi R}{\frac{2\pi r}{T}} = \frac{RT}{r}$$

$$\therefore V.R. = \frac{RT}{r}$$

If handle is used in place of effort wheel,

$\therefore V.R. = \frac{LT}{r}$ Where, L = length of the handle

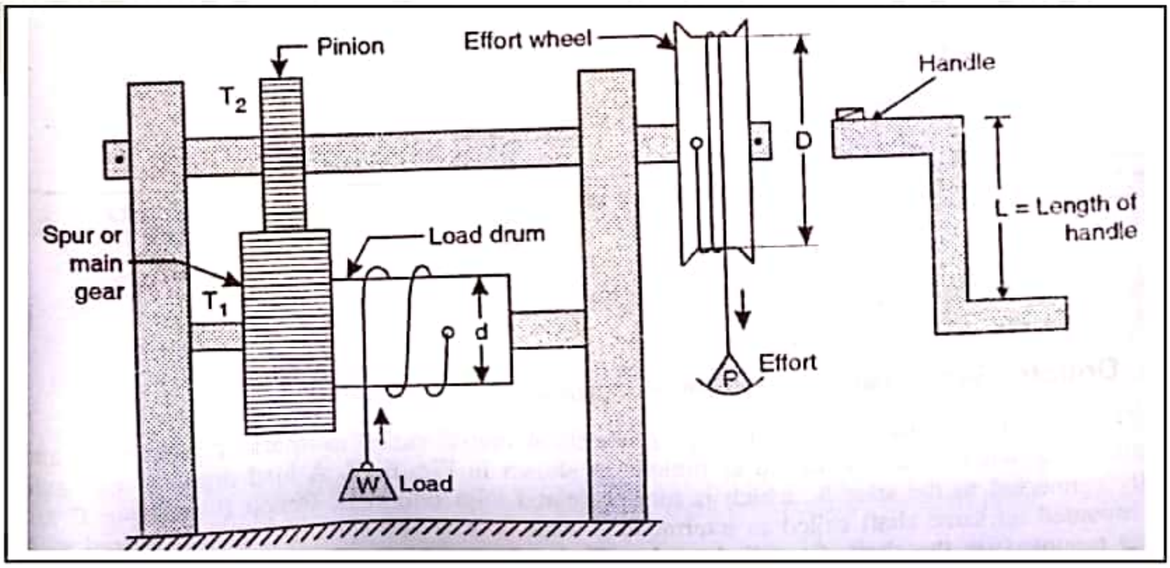
In general, if worm is 'n' threaded,

$\therefore V.R. = \frac{RT}{nr} = \frac{LT}{nr}$ Where, n = number of thread on worm

If the worm is double threaded,

$\therefore V.R. = \frac{RT}{2r} = \frac{LT}{2r}$ since, n = 2

**6. Single Gear Crab or Single Purchase Winch Crab:**




This machine consists of mainly the larger gear wheel known as Spur or main gear and smaller gear known as pinion. Spur or main gear is mounted rigidly on the load drum or load axle and the spur is geared to the pinion which is further mounted rigidly on the shaft to which the effort wheel or handle is attached.

A rope or string is wound round the load drum so as to lift the load 'W' and an another string is wound round the wheel so as to apply the effort 'P' as shown in the figure.

Let, W = Load lifted, P = Effort applied, D = diameter of effort wheel, $T_1$ = number of teeth on spur or main gear, $T_2$ = number of teeth on pinion d = diameter of load drum or load axle.

For one complete revolution of effort wheel, distance moved by the effort = πD and the pinion also completes one revolution, that time spur performs $\frac{T_2}{T_1}$ revolutions and as load drum and spur being rigidly connected, the load drum also performs $\frac{T_2}{T_1}$ revolutions.

$$\therefore \text{Distance moved by the load} = \frac{T_2}{T_1} \text{ x circumference of load drum}$$

$$= \frac{T_2}{T_1} \times \pi d$$

We know that,

$$\text{V.R.} = \frac{\text{Distance travelled by the effort}}{\text{Distance travelled by the load}} = \frac{\pi D}{\frac{T_2}{T_1} \times \pi d}$$

$$\therefore \text{V.R.} = \frac{D}{d} \times \frac{T_2}{T_1}$$

If handle of length 'L' is used in place of effort wheel,

$$\therefore \text{V.R.} = \frac{2L}{d} \times \frac{T_2}{T_1}$$ where, L =length of the handle.

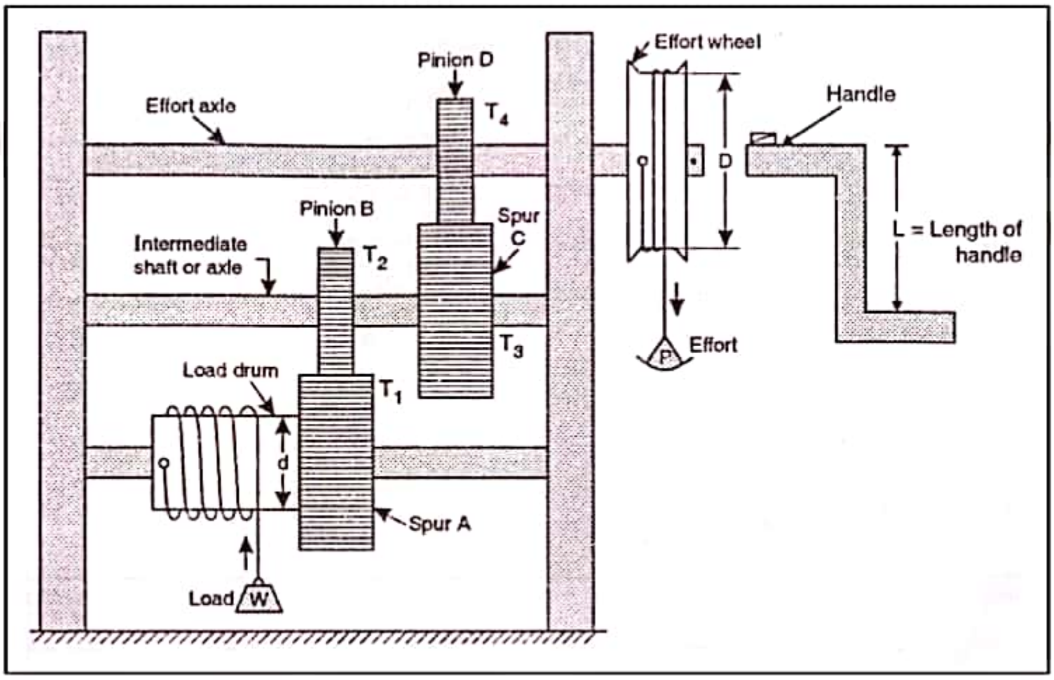
**7. Double Gear Crab or Double Purchase Winch Crab:**

This machine consists of two larger gear wheels A and C called as spurs or main gears and smaller gear wheels B and D called as pinions as shown in the figure. A load drum or load axle is rigidly connected to the spur A, which is further geared with pinion B. Pinion B and spur C are rigidly mounted on same shaft called as intermediate shaft or axle and spur C is further geared with pinion D mounted on the shaft. So called as effort axle to which the effort wheel is attached.

Let, W = Load lifted, P = Effort applied, D = diameter of effort wheel, d = diameter of load drum or load axle.

$T_1$ = number of teeth on spur A, $T_2$ = number of teeth on pinion B mounted on intermediate shaft, $T_3$ = number of teeth on spur C mounted on intermediate shaft,

$T_4$ = number of teeth on pinion D mounted on effort axle




For one complete revolution of effort wheel,

Distance moved by effort = $\pi D$

The pinion D on the effort axle also makes one revolution and therefore spur C and pinion B on the intermediate shaft performs $\frac{T_4}{T_3}$ revolution.

That time, spur A and load drum performs same revolutions as $\frac{T_4}{T_3} \times \frac{T_2}{T_1}$ because both are rigidly connected to each other.

$\therefore$ Distance moved by the load= $\frac{T_4}{T_3} \times \frac{T_2}{T_1}$ revolutions x circumference of load drum $= \frac{T_4}{T_3} \times \frac{T_2}{T_1} \times \pi d$

We know that,

$$\text{V.R.} = \frac{\text{Distance travelled by the effort}}{\text{Distance travelled by the load}} = \frac{\pi D}{\frac{T_4}{T_3} \times \frac{T_2}{T_1} \times \pi d}$$

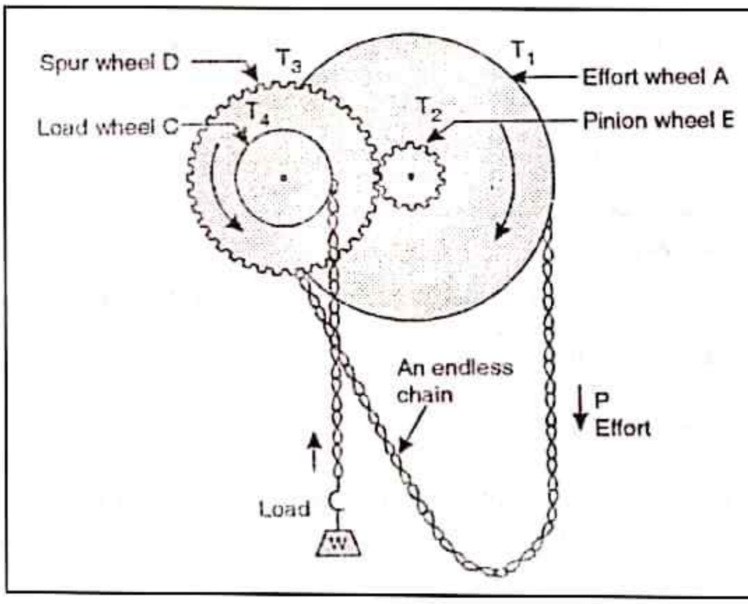
$$\therefore \text{V.R.} = \frac{D}{d} \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

If handle of length 'L' is used in place of effort wheel,

$$\therefore \text{V.R.} = \frac{2L}{d} \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$ where, L =length of the handle.

**8. Geared Pulley Block:**

It consists of effort wheel or cog wheel A and a small gear wheel called as pinion B mounted on the same shaft. It also consists of a load wheel C and a spur wheel D, mounted on the same shaft. The spur wheel D is geared with the pinion wheel B.




An endless chain passes over effort wheel from which effort 'P' is applied. The load 'W' is attached to a chain which passes over the load wheel C as shown in the figure.

Let,

$T_1$ = number of cogs on the effort wheel A,

$T_2$ = number of teeth on pinion wheel B,

$T_3$ = number of teeth on spur wheel or main gear D,

$T_4$ = number of cogs on the load wheel C

For one complete revolution of effort wheel A (cg wheel), we get

Distance moved by effort = $T_1$

Pinion wheel B also completes one revolution and this causes the spur wheel D to rotate through $\frac{T_2}{T_3}$ revolutions.

$\therefore$ Distance moved by the load = $\frac{T_2}{T_3} \times T_4$

We know that,

$$V.R. = \frac{\text{Distance travelled by the effort}}{\text{Distance travelled by the load}} = \frac{T_1}{\frac{T_2}{T_3} \times T_4}$$

$$\therefore V.R. = \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$



If the diameters of effort wheel A and load wheel C are given, then

$$\therefore V.R. = \frac{D}{d} \times \frac{T_3}{T_4}$$

Where, D = diameter of effort wheel A, d = diameter of load wheel C

$T_3$ = number of teeth on spur wheel B,

$T_4$ = number of teeth on pinion wheel D